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Paper Title: **CHILDREN'S CONCEPTIONS OF EQUALS AND ARITHMETIC AND THE NOTION OF NEGATIVE PROCESSING**

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CHILDREN'S CONCEPTIONS OF EQUALS AND ARITHMETIC AND THE NOTION OF NEGATIVE PROCESSING

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Abstract

This presentation will report on two studies: (1) a cross-sectioal study of children's understanding of equals and equivalence; and (2) a longitudinal study of children's proficiency with two-digit mental addition and subtraction. The first study found that children tended to hold a variety of meanings for equals depending on the context in which it was met, and to have a particlar unfamiliarity with the equivalence property of reflexivity and with equals as an operator, The second study identified and classified effective and inneffective mental computation strategies around the three perspectives of approach, process and calculation. In both studies, the children's incomplete and unproficient behaviours can be related to narrow conceptions and inneffective strategies, which in turn can be explained in terms of lack of knowledge and poor thinking processes. However, the two studies provided some evidence that, at times, children may have knowledge and thinking processes adequate for the situation, but not acitivate them because of the action of an inhibitor. This presentation will focus on one form of inhibitor, viz.,negative processing (Cooper, 1987), its role in misconceptions and implications for remedial instruction. It will discuss similar notions, e.g. the critic of Brown & van Lehn (1980), and provide examples of negative processing in children's responses to equals and computation situations.

Studies of children's behaviour may have a focus which includes actions such as describing and categorising children's performance. Reflections on these performance analyses may include such activities as drawing inferences as to the understandings and competencies that lie behind the performance and the construction of theories and models that might explain or illuminate performance. Reflections on reflections may allow, among other responses, theories and models in different contexts, disciplines and discourses to be considered in relation to each other, an activity that may lead to the identification of possible macro-structures that apply across these contexts, disciplines and discourses.

This paper emerges from a period of reflection on reflection that attempted to take a theory (negative power) from one discipline and discourse (sociology and power) and search for a counterpart theory (negative processing) in another discipline and discourse (educational psychology and knowledge acquisition and use). As such, it is a construct that has been developed theoretically, and applied to children's conceptions. It has not emerged from experiences with children and their conceptions and is therefore prone to reflect the idiosyncracies of the applier not the experiences of the application. Thus, it should only be seen as an attempt to provide illumination.

Negative power and negative processing

Cooper, Atweh, Baturo and Smith (1993) have argued that mathematics seems to be central to the social-reproductive activity of schooling. In this, they have supported the conclusions of Stake and Easley (1978) that success in mathematics appears to act as a badge for eligibility for many of the privileges of society and that mathematics teachers recognised this role of mathematics and wanted help in ensuring that their teaching of mathematics reflected appropriate work values. As Cooper et al. (1993) concluded: "... mathematics is a crucial subject for reproducing existing social values and that teachers modify, albeit unconsciously, curriculum material according to the social class and gender of their students".

However, as Walkerdine (1992) has argued, current mathematics curricula tend to have a culture, class and gender bias towards white upper-class males. Evidence of this bias can be found in the studies of Anyon (1981), Atweh and Cooper (1982), Becker (1981), Lorenz (1980), Nunes (1992), and Willis (1990). The consequence of this is that using mathematics as a sorting mechanism builds inequity into the system. Mathematics instruction, therefore, can be seen as an act of power to maintain privilege in society.

This position is based on the critical sociological theory of the seventies that took into account social and political outcomes for education and argued that schools play a primary and direct role in preparing students from different social backgrounds to meet the needs of an unequal society (Bowles & Gintis, 1976). This position was found to be too deterministic and a more complicated picture of how schools reproduce the inequalities of society (Apple, 1981; Giroux, 1980, 1983; Willis, 1977) was developed in the late seventies and early eighties. Students were not found to be passive recipients of the values and knowledge of the dominant culture; in fact, they often resisted and rejected many of the values transmitted through the school.

The basis of the later sociological theories was the notion of hegemony developed by Gramsci (1971). Gramsci used this idea to refer to the set of beliefs, values and attitudes which defines the existing social order as right and legitimate (see Christensen, Gerber and Everhart, 1986, for a good description of hegemony) and argued that hegemonies existed across society and within institutions. However, when this notion of hegemony is added to Bachrach and Baratz's (1976) notion of non-decision, a more subtle explanation for the exercise of power can be developed. Non-decisions are the basis of a second inactive (i.e. negative) face of power, a face which ensures, by limiting the agenda of alternatives considered to be legitimate, that potentially harmful issues and challenges are not raised. By virtue of control of privileges and rewards, a principal, for instance, can wield power in a negative fashion without giving any direct instructions to staff or applying any vetos to staff activity (see Cooper, 1988, for a fuller explanation of negative power). Staff safeguard their present positions and future possibilities by initiating only those actions they know are acceptable to the principal.

The thesis of this paper is that theoretical explanations such as negative power should have application in other discourses and disciplines. For instance, human thinking is often considered as an interaction between existing knowledge, internal perceptions and representations, and external stimuli. As such it appears amenable to ideas from the study of social interactions. Therefore, two faces of processing are posed: the first where unmet goals activate knowledge; and the second, negative processing, where goals limit or inhibit the activation of knowledge. Now we have a new reason for children's different conceptions: learners may have the knowledge to activate and the thinking processes with which to undertake the activation, but not undertake the activation because they have a framework over thinking (an "hegemony" of thought - Cooper, 1988) which inhibit knowledge and thinking as options in the given situation. Such a conception of negative processing would seem to have some similarity to Winston's (1979) censor nodes, Brown and Van Lehn's (1980) critic and Skemp's (1978) and Buxton's (1981) anti-goals. It is also related to the notion of proactive inhibition as described in Dole (1993).

To tease out this notion, particular aspects of two studies of children's conceptions will be analysed in terms of negative processing.

STUDY A: EQUALS AND EQUIVALENCE P-10

Behr, Erlwanger & Nichols (1980) studied 6- to 12-year-old students' understanding of the equals sign in open number sentences. They argued that, to adults, the equals sign in sentences such as 2+4=6 is, intuitively, an abstraction of the notion of sameness and, on a more sophisticated level, an equivalence relation. They further argued that sentences with no plus sign (e.g. 3=3), or more than one plus sign (e.g. 2+1=2+1), do not suggest an action to adults; rather, these sentences are seen to "require a judgement about their truth-value" (p.14). They found that students: (1) understood the equals sign in number sentences such as 2+4= as meaning that something had to be done; (2) did not see 3+2=2+3 in terms of sameness, but rather as an action by restating the sentence as 3+2=5 or 5=2+3; (3) would not accept the equals sign in sentences without it being preceded with one or more operation sign; (4) had "an extreme rigidity" about written sentences and a tendency to perform actions rather than reflect; and (5) did not "change in their thinking about equity as they get older" (p.15).

In light of these findings, a cross-sectional study (Cooper, Rixon & Burnette, 1993) to explore understanding of equals and equivalence was undertaken with students in years P, 2, 4, 6, 8, and 10. It documented the extent of these students' conceptions of same and different, the equals sign, the properties of equivalence, and equals as a relation and an operator.

Subjects

The subjects in the study were chosen by their teachers to represent the range of abilities in their classes and included: (a) six students, four girls and two boys, from a private inner-city preschool; (b) eighteen students from an inner-city private catholic primary school, three boys and three girls from each of years 2, 4, and 6; (c) twelve students from an inner-city private catholic secondary school, three girls and three boys from each of years 8 and 10.

Instrument

The instrument used was the "mixed cases" focused clinical interview, an approach to data gathering based on talk aloud as well as traditional interview techniques. Students were interviewed whilst working on the following four sets of equals and equivalence tasks: (1) the notions of same and different - the students were asked to identify same and different for a variety of topic areas; (2) formal understanding of the equals sign - the students were asked what the sign meant in different situations; (3) the equivalence properties - the students' understanding of reflexivity, symmetry, and transitivity were explored; and (4) static and dynamic approaches - the students' meanings of equals as a balancing relation and as a transformation or change operator were explored. The tasks involved the students working in situations and contexts appropriate to their year level: counters, blocks, pattern cards and drawings in the early years, and equivalent fractions, similar shapes and number and algebra statements and sentences in the later years. Some novel equality situations were considered in the later years.

Procedure

The students were interviewed at their school in a spare room. The interviews were video-taped. The focus of analysis was the ideas of equals and equivalence held by the students in terms of their present mathematical level.

Analysis

The video-tapes were transcribed into protocols and analysed for commonalities. From these, categories of understanding that might explain behaviour were inferred.

Results and discussion

The students' reactions to the four sets of tasks varied. Some tasks were completed with what appeared to be good understanding, including those relating to same and difference (in accord with Fischer & Beckey, 1990) and to the equivalence property of transitivity. Other tasks were not often completed satisfactorily and appeared to be little understood, particularly those relating to the equivalence property of transitivity and to the transformation approach to equals.

However, behaviours were not always able to be interpreted in a straightforward manner. For instance, although the results with respect to same and difference showed that students could identify same and different, in many cases this only occurred under questioning. It showed that, for these students, the idea of considering two things are the same, when this means the same in some attributes while being different in other attributes, in not a natural or easily accepted understanding.

These subtleties in understanding were also evident in meaning of equality. Similar to the literature (Behr, Erlwanger & Nichols, 1980; Ginsburg, 1982; Kieran, 1992; MacGregor, 1991), the equals sign was predominantly seen as an action, e.g. "2+3= leaves something to be done while in 2+3=5 it has been done". This action orientation led to some interesting conclusions from students, e.g. seeing 5=2+3 type examples as being "the wrong way around". However, there were some indications that students see the equals sign as meaning something slightly different in examples such as 2+3=5, 2+3=? and x+3=5. The subtlety in these distinctions appeared to increase with age. For many students, the equals sign in 2+3=5 is seen as the sides being the same while in 2+3=? as a direction to work something out. Older students would often say that the sign means the same thing in all situations but then proceed to assign a different meaning to different situations, e.g. "It is the same, but you have to find the answer in this situation (2+3=?)".

When the equals sign was considered in other situations, e.g. equivalent fractions, similar shapes, the quality of the students' responses appeared to depend on their familiarity with the context. In the novel situation, used only for the older students, where I+I was made equal to XX for the reason that four popsicle sticks (or four straight lines) were used to make either side, students appeared to have real difficulty coping with having an equals sign in this statement.

For the equivalence properties, transitivity appeared to be well understood, symmetry reasonably understood and reflexivity not recognised. When faced with a situation where a design card or a number is placed in a collection and the question is asked to find something in the collection that is the same as that card/number, they would rarely pick up the starting card or number.

Equality relationships were commonly seen in terms of balance and students were skilful in returning relationships to balance when they were unbalanced. Equality as transformation was less familiar and older students appeared unable to interpret examples such as x+3=5 in terms of change and reversing change, even when an example was worked through with them.

STUDY B: MENTAL ADDITION AND SUBTRACTION

Providing instruction in pen-and-paper algorithms early may diminish the use of invented strategies and mental ability and therefore act against the needs of children in a technological society (Sowder, 1990). There are strong doubts as to the need for formal pen-and-paper algorithms at all and support for restricting recording procedures to informal child-chosen techniques (Shuard, 1991).

As a consequence of this, a longitudinal study (Cooper, Herdsfield & Irons, 1992, 1993) was undertaken across years 2 and 3 to study children's spontaneously-derived cognitive strategies for one-, two- and three-digit mental addition and subtraction and the effect of instruction from the traditional pen-and-paper algorithms on these strategies. Its purpose was to describe the range of strategies identified and the changes in strategy use across the two years.

Subjects

The subjects of the study were 65 girls and 65 boys from 3 state primary schools and 3 private primary schools, representing a variety of abilities and socio-economic backgrounds.

Instruments

The instrument used was a 'mixed cases' interview (Ginsburg, Kossan, Schwartz & Swanson, 1983). The children were presented with a sequence of one-, two- and three-digit addition and subtraction tasks to solve mentally. The sequence moved: (1) from addition to subtraction; (2) from one-digit to two-digit to three-digit numbers; (3) from non-renaming to renaming situations; (4) from a real world problem and pictorial presentation form to symbolic presentation form (horizontal to vertical); (5) from take-away to missing addend to comparison concepts of subtraction; and (6) from strategy-friendly to non-strategy examples (e.g. 26+49 to 26+47). The sequence of addition and subtraction examples was designed so that when children began to fail at mental addition, the appropriate level at which to begin the subtraction examples was evident.

Procedure

The children were interviewed six times between the beginning of year 2 and the beginning of year 4 (the period of instruction in pen-and-paper algorithms in Queensland schools). They were withdrawn from their classroom and interviewed in a vacant room within the school. The interviews lasted approximately 20 minutes and were all videotaped.

For the different areas, addition, subtraction, real world and symbolic, the children were given examples of increasing difficulty until they were unable to provide a strategy or unwilling to attempt the example. The interview procedure was designed to challenge, not threaten, the child.

Analysis

The videotapes were transcribed into protocols and the children's solution behaviours categorised. These categories of performance were then related to similar findings from the literature (e.g. Carpenter & Moser, 1984; Ginsburg, 1977) and to analyses of expert behaviour; and competence categories, categories involving the presence of cognitive strategies, hypothesised to explain the behaviours. The analysis focused on findings: (1) across children for the various example types; (2) within children across the different example types; and (3) across the six interviews for individual children and the cohort as a whole.

The analysis is just beginning and results, other than in a general form, are only available for 46 of the children.

Results and discussion

The strategies identified in Interview 1, held before instruction, were as follows. The strategy, COUNT-ON, of counting on the second number (e.g. 2+5 is 2, 3, 4, 5, 6, 7) was not differentiated from the strategy, MIN, of counting on the smaller number (e.g. 2+5 is 5, 6, 7).

(1) Count-all (SUM) - representing the numbers with materials or fingers and counting both numbers, e.g. 4+7 is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 (touching 11 objects);

(2) Count on/count back (objects) - representing the second or smaller number with materials or fingers and counting on or back that many from the first or other number, e.g. 4+7 is 4, 5, 6, 7, 8, 9, 10 and 11 (touching 7 objects) or 4+7 is 7, 8, 9, 10, 11 (touching 4 objects);

(3) Count on/count back (mind) - counting on or back the second or smaller number mentally, e.g. 4+7 is 4, 5, 6, 7, 8, 9, 10, 11 or is 7, 8, 9, 10, 11 (no objects);

(4) Addition-fact thinking strategies (mostly for sums up to 20) - mentally using one of the following:

- *near doubles*, e.g. 4+5 is double 4 plus 1, or 14-8 is 14-7-1;
- *near tens*, e.g. 4+8 is 4+10-2 or 2+10, or 13-9 is 13-10+1 or 14-10;
- *think addition*, e.g. 12-5 is “what + 5 is 12”.

(5) Numeration L->R - dividing one or both of the numbers along place value lines and adding the tens first, as in the three cases below:

- *splitting*, e.g. 13+25 is 10+20 and 3+5;
- *transition*, e.g. 13+25 is 10+20=30+3+5 is 38,
- *aggregation*, e.g. 13 +25 is 13+20 is 33+5 is 38.

(6) Numeration R->L - dividing one or both of the numbers along place value lines and adding the ones first, as in the three cases below:

- *splitting or pen-and-paper approach*, e.g. 13+25 is 3+5 and 10+20;
- *aggregation*, e.g. 13 +25 is 13+5 is 18+20 is 38.

(7) Wholistic - using the numbers as total amounts, e.g. 39+47 is 40+56.

In later interviews, as the examples children attempted became more complex, the categorisations from Interview 1 were unable to classify some of the more sophisticated solutions (e.g. 39+57 is 87+10=97-1) and classified many interview-different solution behaviours in the same category (e.g. 45-26 is 40-20=20-1=19 and 96-39 is 66-10 is 56+1=57). To meet this situation, a more complex categorisation was constructed based around the following three perspectives.

1. *Approach*

This covered how numbers were used and included the sub-categories below:

Approach 1 - simplistic (taking a simplified view of the example, usually wrong),
Approach 2 - partitioned (breaking the numbers into parts),
Approach 3 - place value R->L (lowest place value first),
Approach 4 - place value L->R (highest place value first),
Approach 5 - place value L->R and R->L (mixture of approaches),
Approach 6 - wholistic (e.g. adding 98 in one step), and
Approach 7 - wholistic integrated with other methods

2. *Process*

This covered how the operation process was achieved and included the sub-categories below:

Process 1 - simplistic,
Process 2 - separated place values (38+15 is 30+10 and 8+5),
Process 3 - aggregation (e.g. 38+15=38+5+5+5),
Process 4 - separated place values and aggregation (e.g. 38+25=38+20+5),
Process 5 - compensation-undoing (e.g. 23+98=23+100-2),
Process 6 - compensation-levelling (e.g. 23+98=21+100), and
Process 7 - benchmark (e.g. 41+59=110).

3. *Calculation*

This covered "basic-fact" style strategies and included the sub-categories below:

Calculation 1 - count all,
Calculation 2 - count on and back, in groups (with materials/fingers),
Calculation 3 - count on and back (mentally),
Calculation 4 - counting in amounts of more than one,
Calculation 5 - near doubles,
Calculation 6 - near tens,
Calculation 7 - relating to known fact (includes inverses such as think addition),
Calculation 8 - memory, and
Calculation 9 - easy memory and hard count.

These new categories enabled complex solutions to be categorised in detail. For example, solution 244+359=544+59=600+3=603 can be categorised as place value L->R, benchmark and near tens, while solution 244+359=240+360+4-1=603 can be categorised as wholistic, compensation-undoing and relating to known fact.

Overall the children tended to grow in their use of strategies in the following way: from count all to count on/count back (objects) to count on/count back (mind) to place value L->R (both separation and aggregation) to a mixture of place value L->R and R->L. Wholistic strategies tended to be used when the numbers were appropriate (e.g. 130-49 and 246+99). The aggregation strategy was used more often for subtraction problems presented from a missing addend perspective (e.g. how many has to be added to 68 to make 90 is done by 68+2=70+20=90 giving answer 22). However, many of these missing-addend problems were solved by more straight forward subtraction strategies and procedures. Across the interviews, children tended not to exhibit a great variety of strategies.

Across the six interviews, there was: (1) a marked decrease in the use of the simplistic approach strategy, a marked increase in the two place value approach strategies and an increase in the sophisticated mixed place value and wholistic approach strategies; (2) a decrease in the use of the simplistic process strategy, a marked increase in the separated place values process strategy and increases in the aggregation and compensation process strategies (most notably in the last three interviews); and (3) a decrease in calculation counting strategies (notably from Interview 1 to 2) and an increase in the 'adult' calculation strategy of memory use or direct recall.

At the same time as strategies became more complex, the complexity of problems that children could successfully complete mentally increased. Most children could complete three-digit problems mentally using place value techniques by the end of the interviews.

An interesting aspect of the research was that, according to their teachers, many of the strategies used by children in the interviews were not exhibited by them during school.

National statements in Australia and America and curriculum projects in England are stressing the priority of mental methods over pen-and-paper. The efficient mental procedures or strategies are therefore place value L->R, aggregation and wholistic, not place value R->L (the pen-and-paper strategy). The results show that children in the early interviews who used place value strategies, were more likely to use a L->R procedure. However, by the end of the interviews, after instruction with the pen-and-paper algorithm, nearly all students exhibited both strategy types. From study of the video-tapes, the R->L procedure appeared to be used when students lacked confidence in their ability to complete the item. However, the R->L strategy seemed to cause overload on working memory and the children using it often had to recalculate to give the answer. Aggregation appeared to be the most efficient strategy in terms of mental load.

With regard to interference between strategies, whether the R->L teaching of pen-and-paper algorithms interferes with the efficient L->R the results from Table 1 showed that they were still able to use other strategies. However, if items being considered are restricted to two-digit addition and subtraction with renaming, and the number of students achieving a correct answer is considered, a different strategy-usage pattern appears to emerge, as is documented in Table 1 below for 46 children.

Table 1: Strategy use for correct completion of two-digit items (46 children)

ITEM	STRATEGY	INTERVIEW 1	2	3	4	5	6
Addition	Separation L->R	5	13	7	26	20	11
	Separation R->L	0	7	11	37	22	46
	Aggregation L->R	2	0	2	0	0	0
	Aggregation R->L	0	0	0	0	0	0
Subtraction	Separation L->R	0	2	0	7	4	4
	Separation R->L	0	0	2	4	13	43
	Aggregation L->R	2	0	0	7	4	2
	Aggregation R->L	0	0	0	4	0	4

The results, although only preliminary, show a reduction in correct usage of L->R strategies and an increase in R->L strategies, particularly in the later interviews (when instruction on pen-and-paper had been completed).

DISCUSSION

The negative processing proposal is that, when faced with a problem situation, the children set their problem solver into action to arrive at an answer (see Brown and Van Lehn's, 1980, repair theory). To assist with the problem solving, children have activators and inhibitors. It is the basis of this paper that effective problem solving requires both activators and inhibitors and requires them in balance (this is supported by Brown & Van Lehn, 1980). To have no activators means a computer-like response in that the insufficiency of domain knowledge inherent in a problem situation will prevent the child from giving any response other than to say that they do not have enough information. To have no inhibitors means that there is no control over the range of responses so any response becomes acceptable. Children with poor activators will show a lack of creativity. Children with poor inhibitors will show over-generalisation, lack of permanence of conclusions and inappropriate behaviour.

The theoretical conclusion is that although good thinking may well require innovation, it is also likely to require the maintenance of consistency and logic. For instance, it seems evident that good thinking is knowing when not to change as well as knowing when to change, i.e. inhibition is a necessary part of thinking.. However, thinking will be at risk if, for a particular situation, an inhibitor prevents necessary domain knowledge or necessary strategies from being used. Hence the term negative processing - the exhibition of a misconception by the unnecessary limiting of options.

The two studies discussed have some interesting aspects that seem to involve negative processing. Looking at the studies, there appear to be at least the following types of negative processing: (1) conflict with previous learning - the child has different information already in memory; (2) poor example set - different meanings and actions for different types of examples; (3) restrictive mathematics classroom culture - what is allowed and not allowed in formal mathematics; (4) wrong teaching - deliberate incorrect instruction; (5) lack of confidence - unwillingness to even attempt certain types of problems; (6) disposition - the relatio of mathematics to the learner; and (7) individual - personality traits that inhibit.

(1) *Conflict with existing knowledge:*
The process of proactive inhibition protects the existing knowledge (see Dole, 1993);

(2) *Poor example set:*
An example of this was in Study A. Children were unable to exhibit the reflexive property. If, for instance, for older children, the number seven was shown and then placed in a hoop with 6-1, 5x2, 4+5 and 6, the children would say that there was nothing in the hoop that was the same or equal to the shown card. When the seven was pointed out to them, the children showed that they knew that 7 was equal to 7 but they also showed that their understanding of equals was, in this 'finding' situation, that there hade to be two different representations. The children had the knowledge that same things can be equal, but this knowledge appeared not to be in their agenda of alternatives when they were placed in an action situation of finding something that is equal to a starting statement or object. This contextual limiting of understanding became more evident when examples such as 2+3=5, 2+3=, 5=2+3 and 2+3=6-1 were considered. As the form of the statement or equation changed, so did the meaning of equals.

It is reasonable to assume that this misconception is due to weaknesses in prior teaching, possibly to a paucity in the variety of examples being given. Such a form of negative processing is often evident in shape and space (geometry) - children will show an understanding of straight line for a variety of orientations of lines, yet only be able to draw rectangles and right angles with horizontal and vertical lines.

(3) *Restrictive mathematics classroom culture:*
An example of this was in Study B. There appeared to be a particular negative processing error evident in children's actions with respect to efficient L->R mental strategies. The students would admit to not using this method in any other situations or to hiding its use from the teacher. Many students would change to the more error prone pen-and-paper R->L mental strategy when the vertical algorithm was given. Others would invent a mental strategy for algorithms in the horizontal form even though they had just

exhibited a reasonably efficient pen-and-paper strategy for the vertical form. Students often admit to using different methods in shops. Different contexts (classroom versus real world or interview room), different presentation forms (horizontal vs vertical) and different goals (exercise vs problem) will often give rise to the use of different methods and the complete inability to use previously successful methods. There seems to be a classroom culture that limits what children will even consider . Mental algorithms are one example of mathematical skills children feel are inappropriate, but another is using trial and error methods or materials and diagrams to solve problems ("Oh I could do it that way, but that is cheating").

(4) *Undue emphasis on short term goals:*
There is a particularly important example of this inhibitor in the subtraction algorithm. Many children have been so inculcated with "you can't take 5 from 2" that they reverse the ones in 42-15 to make it 45-12 when faced with this problem

(5) *Lack of confidence:*
Many children appear to be so affected with failure that they will take an area of mathematics and simply refuse to do anything in that area, e.g. division, decimals, algebra.

(6) *Disposition:*
Over time, children can acquire, mostly from the tacit information contained in the didactic relations between them and their teachers, a disposition towards mathematics that prevents them from using their knowledge in certain mathematics discourses and contexts.

(7) *Individual:*
Some children have particular personality traits that become habituated as negative processes. For example, children may not handle the uncertainty of problem solving and may need to have answers immediately. This can lead to an inability to apply knowledge they do have to problems when this application requires time without an answer.

As argued in Cooper (1988), inneffective or unproficient conceptions that arise from negative processing may be particularly difficult to remediate. When an inadequate conception is due to a lack of knowledge or strategies, the remediation seems straightforward - the teacher acts to provide the knowledge and strategies in a way that improves understanding and develops thinking processes. However, when an inadequate conception is due to negative processing, remediation may not be able to be tackled this way. The children may have the knowledge and the strategies but have an orientation to mathematics that precludes this knowledge being used in the situation - the difficulty is not inadequate information but restrictive activation (i.e. inhibitors). Improvement may require the children to recognise and remove the constraints that inhibit thinking, i.e. the children may have to legitimise (make legitimate), for the particular situation, the knowledge that will provide the proficiency. This can be particularly difficult for students who lack confidence.

Present information is lending credence to the idea that legitimising is a metacognitive activity (see Dole, 1993) that requires children to recognise and confront their own limitations with respect to what they believe is legitimate. These beliefs may be deeply, yet tacitly, held and require an act of empowerment.

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