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Error patterns and subtraction knowledge development - a comparison of methods.

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ABSTRACT

Many errors in arithmetical computation are not random or careless; they are learned and have become habitual, and often reveal little about the student's conceptual understanding of the computational principle. There is a need to assist students who exhibit habitual computational errors, as errors reflect the student's lack of meaningful understanding of the computational procedure. The success, however, of corrective instruction is affected by many factors. These include the educator's expertise, the student's prior experiences with learning failure, the student's response to corrective instruction, the nature of the learning difficulty, the accuracy of the error diagnosis, the relationship between the student and the educator, and primarily, the degree of transfer of learning from the corrective setting to the regular classroom.

This paper describes a study which investigated two methods of instruction for correcting systematic computational errors and promoting subtraction knowledge growth in upper primary children. Error analysis enabled consistent patterns of error to be identified, and thus provided details of individual's computational knowledge. This was contrasted to individual's intuitive, concrete and principled/conceptual knowledge (as per Leinhardt, 1988). The two treatment groups experienced contrasting methods of remediation in a clinical setting. The first method, based on systematic and structured reteaching, focussed on tightly linking the symbolic subtraction procedure to the concrete/pictorial representation with use of appropriate language. The second method utilised the Old Way/New Way (O/N) technique, a remediation method developed by Lyndon (1989) based on the psychological concept of proactive inhibition. Lyndon contends that many factors affecting remediation, such as task avoidance and transfer of learning, are attributable to proactive inhibition, which becomes activated when new learning conflicts with prior knowledge.

Results showed that O/N was successful in changing computational knowledge expediently and fairly effortlessly on the part of the researcher, in overcoming factors affecting remediation, such as motivation and avoidance behaviour, and in building concrete and principled/conceptual knowledge, although to a lesser extent than for computational knowledge. In contrast, even with increased effort on the part of the researcher, the

conventional approach proved less successful in improving computational knowledge and only marginally better in building concrete and principled/conceptual knowledge. As a result of this study, it was evident that by incorporating the most promising aspects fo the two instructional techniques, a powerful and time efficient sequence for remediation of systematic errors in subtraction was attained. Remediation attempts beginning with the O/N procedure appeared to lay the foundation for a successful remediation sequence.

BACKGROUND

Mathematical Knowledge

In the past, basic mathematical knowledge was viewed as a student's proficiency in arithmetical calculation (Putnam, Lampert, & Peterson, 1990), and thus traditionally, the teaching of computational skill was dominant in mathematics instruction (Lampert, 1986). The literature, however, provides many descriptions of what mathematical knowledge, or knowing mathematics, is, and it can be seen that computational knowledge is only one component. For example, Leinhardt (1988) has suggested that knowing mathematics derives from four knowledge types: *intuitive, concrete, computational*, and *principled/conceptual*. According to Leinhardt, intuitive knowledge is 'everyday' or real world application knowledge which is normally acquired before formal instruction, concrete knowledge is knowledge associated with representation by appropriate concrete materials during instruction, computational knowledge is the 'this is how to do it' knowledge associated with formal procedures, and principled/conceptual knowledge is the "...underlying knowledge of mathematics from which the constraints can be deduced."(pp. 122) Ginsburg (1977) has described two types of mathematical knowledge, *intuitive* and *formal*. Ginsburg's intuitive knowledge is constructed through children problem solving in their own environment,while formal mathematical knowledge is the result of school instruction. Ginsburg suggested that formal mathematical knowledge is built upon, and links to intuitive knowledge. Resnick (1982) has suggested that mathematical knowledge is both *syntactic* and *semantic*. Syntactic knowledge is correct performance of mathematical procedures, and semantic knowledge is the understanding of the meaning of those procedures. Other knowledge types have been described and labelled along similar lines. Skemp (1978) has described mathematical knowledge as *relational* and *instrumental*; Anderson (1985) has described it in terms of *declarative* and *procedural*; and Case (1982) and Kamii (1985) have discussed knowledge as constructs due to cognitive maturation.

Subtraction Knowledge

The above descriptions differentiate between knowledge types, and then go on to suggest that, as knowledge types can grow and develop in relative isolation to each other, mathematical knowing is the linkage of those knowledge types. Putnam et al. (1990, p. 70) summarise current definitions of mathematical knowing as the "...relationship between the external representation of a mathematical idea and its internal representation in the mind of the learner..." Thus, the knower develops various internalised representations of related mathematical ideas, and easily moves between each representation. Knowing subtraction, then, cannot simply be proficiency in computation. Subtraction knowledge is the melding of principled/conceptual and intuitive knowledge from which computational knowledge is derived (Leinhardt, 1988); it is intuitive knowledge, a child's ability to solve subtraction problems as experienced in real life, providing the basis for formal knowledge growth and the meaningful application of the standard subtraction algorithm (Ginsburg 1977); it is the link of the syntax (the procedures used in the subtraction algorithm) with the semantics (the meaning of the procedures) in the mind of the learner (Resnick, 1982).

Errors

Errors in computation, then, can be regarded as indicative of a student's underdeveloped computational knowledge, or as Leinhardt (1988) has suggested, the absence of linkage between computational knowledge and principled/conceptual knowledge.

Analysis of errors in computation has revealed that many student errors are not careless or random, but occur regularly and consistently (Brumfield & Moore; 1985, Cox, 1975) and, through repetition, have become learned habits. They are produced automatically in response to a stimulus, and in contrast to random, careless errors, are not self-detected nor self-corrected. They are conceptual and learned (Ashlock, 1986).

Errors as Learning Disabilities

Traditionally, students who made errors in their work were regarded as suffering from some learning disability (Kephart, 1960). It was considered that students made errors because they lacked knowledge of the 'correct' algorithm. The implication was that these students required slow and progressive re-teaching so that their knowledge deficit could be repaired. This deficit model of error production assumed that the students remained ignorant about the correct way and that nothing had been learnt as a result of the original teaching effort.

Errors as Learned Disabilities

An alternative to this traditional explanation of error production states that errors indicate the presence rather than the absence of learning. What has been learned, according to this view, is an incorrect way of doing things. Consistency in errors indicates that the student is, in fact, capable of learning but has somehow acquired a learned disability rather than a learning disability (Ashlock, 1986).

Consistent Errors in terms of Bug Theory

A theory to describe how children develop patterns of error was proposed by Brown and Van Lehn (1982). Using computer terms, they called students' errors as "bugs", and labelled the process through which children developed these bugs as Repair Theory. Repair Theory was used to explain the process of how children developed consistent patterns of error. They stated that when learners are confronted with tasks which they are unsure how to perform (on which they have become 'stuck'), they use a simple 'repair' tactic which enables them to produce a solution and become 'unstuck'. In this way, repairs occur as a result of the learner's choosing alternative solution paths in order to produce answers. However, if the repair is erroneous and is left unchecked,the incorrect repair will become a habit, through repetition and practice, to be produced in response to appropriate stimuli. The repair has become a consistent error: a buggy solution. Some students also take several alternative solution paths in response to the one stimulus, hence switching between bugs. This is called bug migration by Brown and Van Lehn.

Factors Affecting Remediation

Without appropriate instructional intervention, systematic, learned errors persist for long periods of time (Cox, 1975). However, the success of corrective instruction is affected by many factors. These include the educator's expertise, the learner's self-confidence, the student's prior experiences with learning failure (fear of failure, task avoidance, 'learned helplessness' - Leder, 1981), the nature of the learning difficulty, the accuracy of the error diagnosis, the student's perception of his/her own mathematical ability, the relationship between the student and the educator, and primarily, the degree of transfer of learning from the corrective setting to the regular classroom (Ashlock, 1986; Bourke, 1980; Covington, 1985).

Proactive Inhibition

It is well documented that some students appear to make satisfactory progress under closely supervised and individualised instruction, but these gains do not transfer to the regular classroom. Regression to the old way of solving a problem is a serious barrier to positive remediation attempts. Although improvement may occur in the short term, these gains appear to fade over time (Read, 1987). Lyndon (1989) has proposed that this observed lack of learning transfer and associated regression to erroneous patterns is due to the mental phenomenon of proactive inhibition (PI). Proactive inhibition is observed when new learning is in conflict with prior learning (Underwood, 1966). To elaborate on proactive inhibition in terms of error patterns and remediation, Lyndon (1989, p. 34) has provided the following summary.

1. Errors represent knowledge, not its absence. It is because children actually know what they are doing that there is a problem with transfer.

2. What the individual knows is protected from change.

3. The protective mechanism is known as proactive inhibition. There is considerable variation within the population in the level of proactive inhibition one inherits. The higher the level of proactive inhibition, the more resistance exhibited towards conventional remediation.

4. Proactive inhibition does not prevent learning from occurring.

5 Proactive inhibition prevents the association of conflicting ideas.

6. Proactive inhibition will inhibit the recall of knowledge which is in conflict with prior knowledge.

Briefly, Lyndon (1989) has argued that consistent errors are protected by proactive inhibition and that proactive inhibition is actually triggered by conventional remediation methods, evidenced by students exhibiting such behaviours as slowness to respond, an apathetic attitude to the task, frustration, and avoidance behaviours. For effective remediation, Lyndon has contended, the remediator must acknowledge PI as an inhibitor of knowledge change and growth, and as such, remediation programs must be structured to effectively deal with proactive inhibition.

Conventional Remediation

For the remediation of systematic errors, approaches incorporating the close linkage of the written representation with the concrete/pictorial representation have been suggested (e.g. Ashlock, 1986; Booker, Irons, & Jones, 1980; Resnick, 1982). However, studies incorporating such methods have revealed that students revert back to their erroneous methods despite the intensity of remediation, and that the lack of positive transfer of new learning and display of avoidance behaviour by the students towards corrective instruction are factors affecting knowledge growth (e.g. Bourke, 1980; Wells, 1982; Wilson, 1982).

Lyndon (1989) has stated that an alternative method, termed Old Way/New Way (O/N), deals with such difficulties in remediation. He states that "...the inhibitory effects of proactive inhibition may be reduced by the use of the O/N method; and use of O/N may lead to the retroactive inhibition (i.e. forgetting) of the "old" knowledge."(p. 34). Thus, O/N, rather than taking a "bottom up" approach to remediation (i.e. reteaching procedures according to good teaching models to enhance learning and knowledge development) can be regarded as a "top-down" approach where erroneous procedures/algorithms are the first point of focus.

Old Way/New Way Methodology

The O/N procedure is based upon bringing the learner's 'old way' to a conscious level and exchanging it for a 'new way' by means of discrimination learning, followed by practice with the correct 'new way'. A simplified example of how the O/N method proceeds through the four steps is provided for the remediation of a systematic error in the subtraction algorithm. In step 1, *reactivation of the error memory*, the student is asked to complete the subtraction problem 306 - 149 in their usual way. For step 2, *labelling and offering an alternative*, the student is asked if that particular method of performing that computation can be called the 'old way'. When consent is given, the student is asked if a 'new way' for computing 306 - 149 can be shown. Using carefully selected language, the remediator performs the algorithm the standard way. The difference between the two algorithms is then carefully pointed out. In step 3, *discrimination*, the student is asked to perform the computation the old way, then the new way, and then asked to contrast the two ways. This discrimination of the same problem (306 - 149) is repeated five times. For step 4, *generalisation*, the student is provided with six subtraction exercises and asked to complete using the new way. This sequence of four steps is called a learning trial, and takes approximately 10 minutes. According to Baxter & Lyndon (1987) the benefits of the O/N method are thus:

O/N bypasses proactive inhibition and enables the remediator to change the child's knowledge base rapidly and permanently...The more or less instantaneous success the child experiences after one trial ensure that avoidance learning behaviours are soon eliminated. Confidence in ability to learn is restored. (p. 8)

Implications for this study

Systematic patterns of error in subtraction computation would be learned habits, produced in response to an appropriate stimulus (Lyndon, 1989). As habits, they would be protected from change by the proactive inhibition mechanism. Further, the error would not be a meaningful application of subtraction within the base ten numeration system. Hence, an individual's computational knowledge would not be linked to intuitive and/or principled/conceptual subtraction knowledge in a meaningful way.

In this study, two methods of remediation were utilised in an attempt to change Year 7 students' (age 12-13 years) erroneous computational subtraction knowledge, and link it to principled/conceptual, intuitive and concrete subtraction knowledge. One remediation method was a conventional sequence based primarily on suggestions by Booker, Irons, & Jones (1980); the other method was Old Way/New Way.

The purposes of this study were:

1. To compare the effectiveness of two methods of remediation in changing Year 7 students' erroneous computational subtraction knowledge, and linking it to principled/conceptual, intuitive and concrete subtraction knowledge.
2. To document subjects' responses to the two methods of remediation.
3. To search for evidence of proactive inhibition affecting remediation.
4. To explore the potential of O/N technique as a tool for mathematics remediation.
5. To hypothesise an effective method of remediation for use with upper primary students.

METHOD

Sample

From a pool of 60 Year 7 students, 16 students were selected for the study. The sample of 16 selected students attended a suburban Brisbane primary school. Subjects were selected upon demonstration of systematic errors in subtraction computation identified via a diagnostic error analysis test.

Measures employed

<u>Subject Selection Instrument</u>

The researcher-made diagnostic error analysis test consisted of five types of subtraction problems classified according to level of computation skill required (see Table 1).

Table 1
Subtraction algorithm skill levels of diagnostic test instrument

Skill Level	Skill	Example
Level A	Subtracting a two-digit from a two-digit number with regrouping	53 -14
Level B	Subtracting a two-digit number from a three-digit number with renaming in ones and tens place	523 - 78
Level C	Subtracting a three-digit from a three-digit number containing zero in the ones place with regrouping in the tens place	260 -156
Level D	Subtracting a three-digit from a three-digit number containing zero in the tens place with renaming in the hundreds place	608 -134
Level E	Subtracting a three-digit from a three-digit number containing zero in the tens place with renaming across the tens to the hundreds place	302 -158

The test contained five problems from each level of computational skill, presented in random order. Test performance was scored by examining errors for the existence of a pattern. For any given skill level, a systematic error was defined as one which occurred three or more times out of five attempts (Cox, 1975).

Interview Schedule

Structured clinical interviews (Ginsburg, 1981) contained items relating to concrete, intuitive, and principled/conceptual subtraction knowledge and results from the diagnostic error analysis test enabled concrete knowledge to be determined. For concrete knowledge, subjects were required to perform the subtraction algorithm using Base 10 (MAB) blocks. For intuitive/real life knowledge, subjects had to (a) solve a real world subtraction problem, and (b) create a real world subtraction problem. For principled/conceptual knowledge, subjects had to demonstrate understanding that (a) by increasing/decreasing the minuend or subtrahend in a subtraction exercises, the solution alters accordingly, and (b)addition is the inverse of subtraction. They alsohad to show (c) evidence that they understood the regrouping/renaming process used for decomposition subtraction algorithms, and (d) an understanding of place value.

PROCEDURE

The selected subjects were randomly assigned to two treatment groups. Each group consisted of students who exhibited consistent errors in all skill levels of subtraction algorithms, and students who exhibited consistent errors in particular skill levels only.

Upon selection and assignment of subjects to treatment groups, the study proceeded via a five step plan.

Step 1-Interview 1

All subjects were individually interviewed prior to treatment, and data on intuitive, concrete, computational and principled/conceptual knowledge for each subject were tabulated.

Step 2-Treatment

Remediation method 1 (used with group 1 subjects) consisted of a series of lessons which centred upon using appropriate materials to demonstrate the legality of the subtraction process within the base ten numeration system. Group 1 subjects experienced 10 lessons of approximately 20 minute duration beginning with regrouping activities using bundling sticks and MAB for the subtraction of single digit numbers from two digit numbers, and progressing through the five skill levels of algorithms used on the diagnostic error analysis test (see table 1). The numeral expander was used to reinforce the regrouping process, and to reduce manipulation of concrete materials for subtraction of large numbers. Although the lessons followed a sequential framework, each lesson was contingent upon progress in the previous lesson. Thus, remediation method 1 followed the format of a teaching experiment (Kantowski, 1978). The 8 subjects in this treatment were withdrawn from the bulk of the class and treatment occurred on 10 consecutive school days.

Remediation method 2 (used with Group 2 subjects) consisted of performing one O/Ntrial individually with each subject. Consistency of error with certain skill level algorithms determined the algorithm used for the O/N trial. A subtraction problem involving a skill level E algorithm was used with subjects exhibiting consistent patterns of error in skill level E algorithms. A subtraction problem involving a skill level B algorithm was used with subjects exhibiting consistent error patterns in all algorithm types as a starting point for O/N. The rationale for this starting point choice was that level B algorithms appeared to be the second most frequently occurring error pattern next to skill level E algorithms. This remediation sequence was also a type of teaching experiment as subsequent O/N trials could only be planned once the need had been determined. Prior to the first round of O/N trials with individual subjects, it was anticipated that further O/N trials would have to be performed with students exhibiting errors in skill levels other than level B algorithms. Each O/N trial took approximately 10 minutes. The day after the O/N trial, Group 2 students met as a whole group, and were presented with 5 skill level B and 5 skill level E exercises. Subjects were expected to complete the exercises relating only to the skill level that had been targeted using O/N. From this information, the experimenter could ascertain which students required further Old Way/New Way trials for other skill level algorithms. Further O/N trials were not required, however. During the two week treatment time allocation, Group 2 met together on one other occasion for a skill maintenance check, where 10 subtraction exercises were presented for computation.

Field notes were gathered during the remediation sessions, and at the conclusion of each session, these notes were translated into detailed summaries of the session's events. Because of the strong contrast between the two remediation methods, of particular interest were subject responses and attitudes to the remediation approaches. Strategic questioning and prompting occurred during the remediation sessions in the hope of eliciting attitudinal responses. Whenever students demonstrated obvious enjoyment in the task (such as enthusiasm to perform, playful bantering between friends, attentiveness to instructions) or obvious resentment towards the task (such as scowling, mumbling, reluctance to perform) subjects were prompted to express their feelings and elaborate on what they felt about they task they were engaged in. In this way, subjects' responses to the two remediation methods could be documented, and hence the effectiveness of methods of remediation for use with upper primary students could be determined.

<u>Step 3-Interview 2</u>

Upon completion of treatment, a second interview, similar to the first, was conducted and data tabulated against data from the first interview. This enabled growth of subtraction knowledge in relation to the four knowledge types to be determined for each subject.

<u>Step 4-Further treatment</u>

In this study, O/N was utilised for the express purpose of changing erroneous computational knowledge. As such, it was anticipated that further instruction would be required to build and link computational knowledge to the other knowledge types for group 2 subjects. Conversely, it was intended that O/N could be used with any group 1 subjects who still exhibited systematic computational errors after remediation method 1.

The purpose of the fourth step of the study then, was an effort to promote all subtraction knowledge types for all students. The results of the second interview were called upon to determine which students required further remediation in terms of the four knowledge types. O/N was used for computational knowledge development, and aspects of the conventional method were employed for concrete and principled/conceptual knowledge development. Step four, then, was the provision of specific programs on the basis of individual need.

Step 5-Interview 3

To determine the appropriateness of computational, concrete, intuitive, and principled/conceptual knowledge for all subjects at the conclusion of the remediation program, a third interview was performed.

RESULTS

Upon completion of treatments offered in step 2, subjects in both groups demonstrated growth of subtraction knowledge. Figures 1 and 2 graphically display the number of subjects from each treatment group who demonstrated appropriate computational, concrete, intuitive, and principled/conceptual knowledge prior to, and upon completion of, treatment.

From Figure 1, it can be seen that for group 1, computational knowledge increased for 4 subjects, concrete knowledge increased for 5 subjects, intuitive knowledge increased for 5 subjects, and principled/conceptual knowledge increased for 6 students. These results show a fairly evenly spread growth of the four knowledge types. Treatment time for each group 1 subject totalled approximately 200 minutes.

For group 2, the O/N method (remediation method 2) was employed to specifically target inappropriate computational knowledge. After one O/N trial, all 8 subjects in group 2 demonstrated appropriate computational knowledge, and growth of other knowledge types was also evident (Figure 2). Concrete knowledge increased for 3 subjects, intuitive knowledge increased for 3 subjects, and principled/conceptual knowledge increased for 5 subjects. These results show growth in the four knowledge types with an emphasis on computational knowledge growth. Total treatment time for each group 2 subject was approximately 20 minutes.

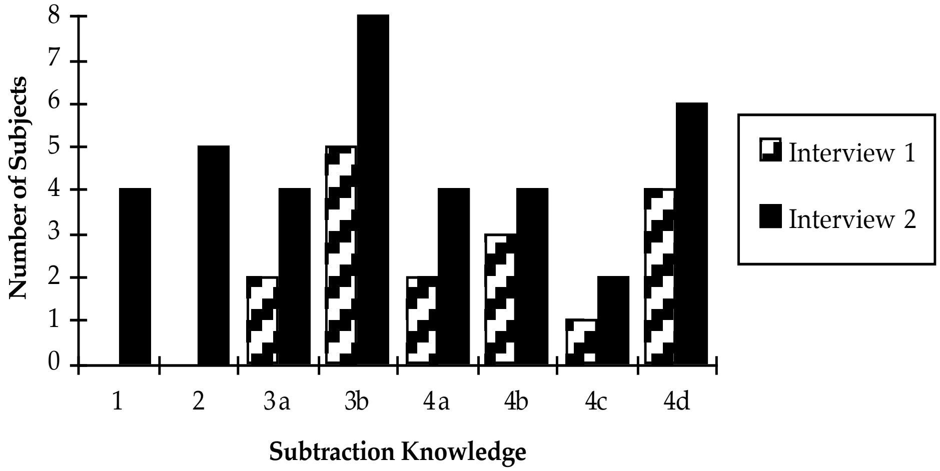

Figure 1. Number of students demonstrating computational, concrete, intuitive, and principled/conceptual subtraction knowledge - Interviews 1 & 2 - Group 1.

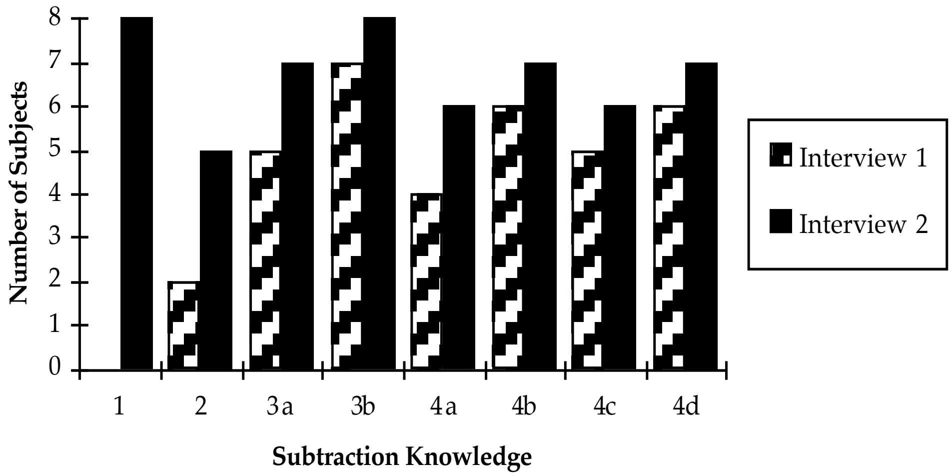

Figure 2. Number of students demonstrating computational, concrete, intuitive, and principled/conceptual subtraction knowledge - Interviews 1 & 2 - Group 2.

Subtraction Knowledge

1. Computational	4. Principled/conceptual
2. Concrete	a) the operation of subtraction
3. Intuitive	b) addition is the inverse of subtraction
a) solving	c) renaming with understanding
b) creating	d) place value

The subject selection test identified subjects who exhibited systematic patterns of errors in subtraction computation, and, as discussed previously, assignment of subjects to treatment groups was performed on the basis of the error patterns. Upon completion of the interviews, it was obvious that group 2 subjects overall had a greater subtraction knowledge than group 1 subjects. Results then, can only be interpreted with caution. Comparison of the two remediation methods can more appropriately be made on the basis of qualitative data relating to the affective domain of the subjects. The following descriptions are a summary of qualitative data gathered during the study.

Remediation method 1 - group 1 subjects

Prior to commencement of remediation method 1 activities, the mood of the group 1 subjects appeared to be positive and the students appeared excited. Comments noted included:

> "Are we going to have some fun?"
>
> "What are we going to do? Make something?"

The students indicated that they had never used materials such as bundling sticks or base ten blocks for subtraction, and they readily engaged in bundling and unbundling groups of ten sticks, and exploring the value of the various base ten blocks. Initially when modelling simple subtraction tasks, certain subjects frequently called out the solutions whilst simultaneously seizing the corresponding number of bundling sticks. Taking this behaviour as a cue, other students then imitated the action of the 'faster' students. When all subjects were reminded their task was to demonstrate the process of attaining the solution rather than the solution itself, behaviours noted included a pounding/striking action for placing the MAB materials on the table, the physical distancing from the main group, the slow and deliberate placement of the blocks onto the place value chart, and the constructing of towers with the blocks rather than completing set tasks. Comments recorded included the following:

> "This is babyish."
>
> "I feel a bit daggy doing this."
>
> "Oh, this is cinch."

Throughout this session and subsequent sessions, the researcher continually had to reiterate that the object of the exercise was not to be the first person to *produce* the correct answer; rather it was to concentrate upon the *process* of reaching that solution. As the sessions progressed to the symbolic recording together with the corresponding manipulation of concrete materials, many subjects disregarded the concrete model and calculated the solution using their own (upon inspection, erroneous) computational methods. The researcher continually reminded subjects of the procedure required; that they work in a step-by-step manner, manipulating the concrete model and then recording this in the symbolic form. The following comments were recorded:

> "Oh, why do we have to do it with blocks?"
>
> "This is so basic."
>
> "I don't need to use blocks. I know how to do this."

When the numeral expanders were introduced, time was spent exploring the way this particular aid related to concrete regrouping as subjects in the group had not previously seen or used such a device. The numeral expander was utilised as an aid for regrouping large numbers in subtraction, and as a transition from concrete to total pen and paper computation. In a one-to-one situation with the researcher, each subject demonstrated appropriate pen and paper computation with the assistance of the numeral expander. However, as subjects worked on their own, numeral expanders were set aside and systematic computational errors were produced by some students.

During one of the final remediation sessions, subjects were required to complete a small worksheet of subtraction exercises. Bundling sticks, base ten blocks, and numeral expanders were set out for use if required, and subjects were instructed to work at their own pace. The following comments from various subjects were recorded in response to this request.

> "I know how to do these. I want to go back and work with the rest of the class."
>
> "I'm useless at these."
>
> "Oh, I don't mind. I want to do these."
>
> "Ugh...I'd rather be doing what the others are doing."

Inspection of the completed worksheets revealed that some students were still performing the algorithm incorrectly, despite apparent positive gains observed during the one-to-one situation with the researcher in previous sessions. From comments and observed body language (such as scowling, groaning, staring out of windows, leaning back on chairs, slowness to respond to required tasks) it was apparent that enthusiasm for these sessions had waned for some students in the group, despite the fact that errors were still being made.

Throughout the sessions, some subjects worked steadily at set tasks, but often had to be called back to attention after being distracted by the physical and verbal protestations made by other subjects.

Remediation method 2 - group 2 subjects

In the first step of the O/N method, students were presented with a multi-column subtraction exercise of one of the skill level algorithms. The following expressions relating to anxiety about mathematics in general, or the subtraction exercise in particular were noted:

"I can't do these, I always get them wrong."

"I'm no good at Maths."

"Oh, I hate these...with the zero. I always get them wrong."

From their comments and body language (slowness to take up the pencil, leaning back in the chair, distancing self from the table) subjects appeared reluctant to perform the algorithm. Once the O/N trial had been completed, the following comments were noted:

"Oh, that's good. Now I know how to do it. Good."

"Oh yeah. I just used to forget about the zero."

"Oh this is easy. I know how to do this now."

After one Old Way/New Way trial, one subject asked whether he could take his work home to show his mother.

After completion of one O/N trial with group 2 subjects, all subjects appeared keen to remain together as a group rather than join the bulk of the classroom. Motivation amongst this group appeared high, and as such the opportunity to work on concrete and principled/conceptual knowledge presented itself. Exemplifying the legitimacy of the subtraction algorithm with concrete materials, exploring the use of addition to check subtraction calculations, linking subtraction to the real world, and developing estimation skills to approximate answers appeared to be the next logical steps in the remediation process.

In step 4 of this five-step study, subjects were no longer treated as two separate groups; rather treatments were administered upon an individual subject needs basis. However, for ease of identification of subjects and interpretation of results, group 1 and group 2 labels were retained. Knowledge growth was determined through Interview 3. For group 1, computational knowledge increased for 2 more subjects, intuitive knowledge increased for 1 more subject, principled/conceptual knowledge increased for 9 more subjects. For group 2, concrete knowledge increased for 3 more subjects, principled/conceptual knowledge increased for 5 more subjects; the numbers of subjects demonstrating computational and intuitive knowledge were unchanged from Interview 2.

At the conclusion of this study, two of the eight group 1 subjects still did not demonstrate appropriate computational knowledge. The O/N technique was used with three of the four group 1 subjects in step 4, and appeared to aid computational knowledge growth for two of those three students. The third subject, when being tested on his 'new' subtraction knowledge, produced a different systematic error to the one targeted using O/N. O/N was not used with the fourth subject due to the failure of the subject to recall her erroneous procedure during the allocated remediation time.

Qualitative data gathered during further treatment provides for documentation of subjects' responses to the two methods of remediation. When O/N was used with group 1 subjects in Step 4, the subjects worked through the procedure quickly and appeared interested in the process. At the generalisation stage, the subjects appeared to be pleased with the fact that they had correctly calculated the six problems presented. The subjects sat up in their chairs and leaned forward over their page as if eager to complete each problem. Upon asking if they would be able to use the new way for subtraction the next day, the following comments were forthcoming:

> "Oh yes, because you've shown me the new way now."
>
> "Yes, I know how to do it properly now."

Selected activities from remediation method 1 for developing concrete and principled/conceptual knowledge were well accepted by the students. All group 2 students appeared interested in exploring the way bundling sticks and base ten blocks could be used to model the pen and paper computation procedure they had learned using the 'new way'. Avoidance behaviours were not evident, and when simple subtraction exercises were used, the students did not have to be constantly reminded to explore the *process* of solution attainment rather than the *product*. The subjects were interested in the numeral expanders and appeared fascinated in looking at the way large numbers could be expanded and regrouped.

DISCUSSION

One of the purposes of this study was to compare the effectiveness of two methods of remediation in changing erroneous computational subtraction knowledge, and linking it to principled/conceptual and concrete subtraction knowledge. Results showed that O/N was successful in changing computational knowledge for all students, and in building concrete and principled/conceptual knowledge, though to a lesser extent than for computational knowledge. The conventional approach, 100 times more time intensive compared to remediation method 2

(O/N), was less successful in improving computational knowledge and only marginally better in building concrete and principled/conceptual knowledge.

Other purposes for this study were to document subjects' responses to the two methods of remediation, to search for evidence of proactive inhibition affecting remediation, to explore the potential of O/N as a tool for mathematics remediation, and hypothesise an effective method of remediation for use with upper primary students. The following discussion attempts to address these purposes of the study.

Comparison of the two remediation methods

Remediation method 1 proved to be a complicated and effortful exercise with Group 1 subjects. The limitations of this remediation method can be summarised as (1) the inability of the lessons to cater to the ability levels of the subjects; (2) the difficulty of controlling the desire of all students to *produce* answers rather than analyse the *procedure* used to gain the answer; (3) the difficulty of maintaining motivation to enable all tasks to be completed; (4) the apparent difficulty in translation of the concrete process to cognitive structure, and to the pen and paper procedure, evidenced by the discarding of base ten materials and numeral expanders at the earliest convenience; (5) the time and energy requirements demanded from all parties to implement this method; and (6) the inability of this method to confront subjects' existing knowledge, as evidenced by the recurrence of systematic errors.

Upon completion of remediation method 1 growth of various subtraction knowledge types for certain subjects was evident. The strength of remediation method 1 was its holistic nature. The approach incorporated real world examples, and the symbolic algorithm could directly be modelled using concrete materials. The approach also leant itself to consistency of language.

The O/N methodology can be regarded as an apparently convergent remediation approach which, in isolation, focuses merely on computational knowledge. This is its apparent weakness. However, in this study,O/N's superiority lay in the short amount of time and effort required for implementation, its power to motivate students, and its ability to confront the effect of proactive inhibition as recurrence of erroneous computation procedures were not evident.

Evidence of proactive inhibition affecting remediation

Many factors appeared to affect the success of remediation method 1, and these findings lend support to Lyndon's (1989) argument on the influence of proactive inhibition and remediation. As previously stated, Lyndon has suggested that children revert to their old patterns of error after remediation attempts because the proactive inhibition mechanism acts in such a way as to prevent the association of conflicting knowledge. In this study, it could be argued that the subtraction procedure demonstrated by the experimenter in remediation method 1 was in conflict with each subject's personal knowledge of the subtraction procedure; knowledge that had become habitual and automatic. As such, several subjects regressed to their old patterns of error when not monitored by the researcher. As Lyndon (1989, p. 34) has stated:

...errors represent knowledge, not its absence; it is because children actually know what they are doing that there is a problem with transfer...Proactive inhibition prevents the association of conflicting ideas...Proactive inhibition will inhibit the recall of knowledge which is in conflict with prior knowledge..."

It appeared that, for some group 1 subjects the activities presented did not link to their other previously learned subtraction knowledge

Lyndon (1989) has claimed that O/N is designed to by-pass the proactive inhibition mechanism, and change the child's knowledge base rapidly and permanently. From the results of remediation method 2, there was no evidence of regression to erroneous procedures in the short term; with remediation method 1 however, there was. Further, Lyndon (1989) has also argued that the proactive inhibition mechanism is triggered by conventional remediation methods, evidenced by students exhibiting avoidance behaviours such as slowness to respond, an apathetic attitude to the task, frustration, and so on. In method 1, such behaviours were noted and discussed as weaknesses of the method with subjects in this study. In method 2, such behaviours were not in evidence. This finding lends support to Baxter and Lyndon's (1987, p. 8) claim that "...the more or less instantaneous success the child experiences after one (Old Way/New Way) trial ensure that avoidance learning behaviours are soon eliminated. Confidence in ability to learn is restored."

Mathematics remediation and upper primary students

The two remediation approaches attacked subtraction computation from entirely different angles. O/N looked first at the error, and was concerned with replacing the erroneous procedure with a correct procedure. Remediation method 1 aimed to use concrete materials to legitimise the steps in the algorithm. From the results in this study, it appeared that O/N provided an excellent starting point for remediation with students of Year 7. Students displayed confidence in their ability, and a perceptible sense of relief at finally being shown the correct way. Once the subtraction algorithm was correctly performed, these students appeared ready to engage in activities designed to develop other aspects of subtraction knowledge.

In contrast, remediation method 1 appeared to be a ‘tiresome’ approach for use with students of this level. Towards the end of the ten lesson sequence, interest in the activities began to obviously wane. The gains in knowledge compared to the amount of effort exerted by the researcher and the students appeared minimal against gains from using O/N.

CONCLUSION

Two purposes of this study were to compare the effectiveness of two methods of remediation, and to hypothesise an effective method of remediation for use with upper primary students. The results of this study suggested that O/N method appeared to be a powerful mechanism for overcoming barriers influencing knowledge growth. The O/N method provided motivation, success, and restored confidence in the individual’s own ability to learn. The structured and sequential activities incorporating the use of concrete materials appeared to be an effective means of promoting concrete, computational and principled/conceptual knowledge, and melding these knowledge types holistically once erroneous computational procedures had been eliminated.

To hypothesise an effective method of remediation for use with upper primary students based on these results, the combined teaching sequence for systematic errors in subtraction would be:

1. Identification of systematic errors.
2. Structured interview to establish depth of intuitive, concrete, and principled/conceptual knowledge.
3. Perform O/N trial to remediate systematic computational errors.
4. Use MAB and other materials to link computation knowledge to concrete knowledge and thus legitimise the algorithm.
5. Through discussion and application, develop approximation and estimation skills, develop checking skills, and build up intuitive knowledge of subtraction in relation to the real world.

Further studies with larger samples and with a wider range of systematic errors in computation will contribute to the generalisability of these results.

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