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Development of Alternative Conceptions of Motion: A Comparison of Pupils' Responses in Three Countries

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ABSTRACT

A systematic study of children's ideas on motion was carried out in three countries: Israel (N=631), England (N=383) and Australia (N=357). An open ended questionnaire with four questions about motion was administered to intact classes of pupils from Grade 2 through Grade 12 (ages 7 to 18). The responses were categorized according to level of sophistication. For three of the questions, it appears that children pass through distinct, successive stages with respect to their conceptual understanding. A mathematical model was developed which gives the proportion of children in each stage as a function of age. It predicts that the proportion of subjects at each stage is a linear combination of decreasing exponentials, and it fits the data well.

INTRODUCTION

How do children's ideas about the physical world arise, and how do they develop as the children mature? These are not new questions; indeed, they are at the heart of Piaget's work on genetic epistemology (see, e.g. Piaget, 1971). In recent years, researchers in science education have renewed the study of these questions from a somewhat different point of view: that of the improvement of science education. Some researchers have emphasized the persistence of naive ideas even after formal education in science (Whitehead, 1991; Westbrook and Marek, 1991, 1992) while other researchers have been most interested in the changes in children's conceptions

as they mature (Stavy, 1988; Bar, 1989; Bar and Travis, 1991; Kuiper, 1991). Eckstein and Shemesh (1993) studied the responses of Israeli children in grades 2 to 12 (ages 7-8 to 17-18) to a four-part questionnaire on different aspects of motion. They found that for three of the questions, the responses change as the children mature, and it appears that children pass through distinct, successive stages with respect to their conceptual views. The results were analyzed with the help of a mathematical model which had first been applied to the development of Piagetian stages (Eckstein and Shemesh, 1992a, 1992b). This model predicts that the proportion of subjects at each stage is a linear combination of decreasing exponentials. The data were well-fitted by the model.

In this paper we report the results of a cross cultural study of children's ideas of motion. In this research, we used the instrument of Eckstein and Shemesh (1993), to investigate the conceptions of children in England and Australia in grades 2 to 11 (ages 7-8 to 16-17). Our research questions were:

1. Are the same stages in the development of the concepts of motion apparent among children in Australia and England as in Israel?

2. What are the differences and similarities between the development of the concepts of motion in the three countries?

3. Does the theoretical model of Eckstein and Shemesh (1993) fit the responses of children in Australia and England as well as those in Israel?

QUESTIONNAIRE AND CATEGORIZATION

The questionnaire consisted of four items, each of which dealt with a different aspect of kinematics, namely, projectile motion on earth, projectile motion in space, relative motion, and circular motion.

Question 1. Projection motion on earth

This was a two-part question. In the first part, a drawing of a ball on a table was displayed with the following instruction:

> 1a. A ball is moving on the table in the direction of the arrow. Draw the path the ball follows after it reaches the edge of the table.

In the second part the same drawing was again displayed, with the following instruction:

> 1b. The ball in this drawing is going faster than the first ball. Draw the path the ball follows after it reaches the edge of the table.

The responses of the subjects were categorized into three main categories, which were thought to be of increasing sophistication, I, II and III.

Category I. Both the slower and the faster ball falls straight down from the edge of the table. Subjects responding in this category have the `commonsense' idea that objects that are not supported fall straight down. This is the `law of support' described by Ogborn (1985), Bliss, Ogborn and Whitelock (1989) and Whitelock (1991).

Category II. The slower ball falls straight down, but the faster ball moves forward. Subjects in this category realize the importance of at least two variables: gravity and velocity.

Category III. Both the slower and faster balls move forward. For subjects in this category, the `law of support' is not valid; they realize that forward motion does not disappear when objects lose their support.

Question 2. Projectile motion in space.

In this question, the same drawing was presented as in Question 1, with the following instruction:

2. This time the ball is moving on a table in a spaceship in space. Draw the path the ball follows after it reaches the edge of the table.

The three main categories of response were:

Category I. Subjects are not aware of the difference between motion on a spaceship and motion on earth.

Category II. Subjects know that motion in space differs from motion on earth, but do not know the precise nature of this difference. Some subjects think that the ball *rises* at the edge of the table. Others show a fluttering, erratic motion.

Category III. Subjects give the correct answer, namely that the ball moves forward in a straight line.

Question 3. Relative motion.

A drawing was displayed of a cart. A small metal ball is held by a magnet at the top of a post on the cart, and a cup is on the floor of the cart directly below the ball. The instructions were as follows:

In this drawing there is a cart that is moving in the direction of the arrow. A small metal ball is held by a magnet to a post on the cart.

3a. Suppose the ball falls off the magnet while the cart is in motion, and the cart keeps on moving with the same speed. Where will the ball fall -- behind the cup, in

front of the cup or in the cup? Please explain your answer.

3b. Another time, the ball falls off while the cart is in motion, but this time the cart stops while the ball is still in the air. Where will the ball fall--behind the cup, in front of the cup or in the cup? Please explain your answer.

The three main categories of response were:

Category I. Responses in this category gave the correct answer to Question 3a, that the ball would fall in the cup, but their explanation, if any, was not related to the correct explanation, namely that the horizontal velocity of the ball was the same as that of the cart. This was called the "intuitive" response by Eckstein and Shemesh (1989).

Category II. Responses in this category were `logical' but incorrect, namely that the ball would fall behind the cart, because the ball falls straight down while the cart continues moving forward.

Category III. Responses in this category were that the ball would fall in the cup for situation 3a, and would fall in front of the cup for situation 3b; in addition, the student's explanation had to make some explicit mention of the forward velocity of the ball (although scientific language was not required).

Question 4. Circular motion.

A drawing of a rolled up hose with one end extended vertically was displayed. The instructions were:

4. Someone rolled up a hose and left it lying on the floor. A pupil raised one end of the hose and threw a ball through it. The ball rolled through the hose and came

out the other side. Draw the path of the ball *after* it left the hose.

This is similar to a question posed by McCloskey, Caramazza and Green (1980) to a group of university students, and it probes conceptions of circular motion. Approximately half of their respondents believed that the path of the ball after leaving the hose would be circular. This is a misconception resulting from a belief in circular impetus. The categories were:

Category I. Backward motion of the ball, non-logical responses and no answer.

Category II. A curved path for the ball.

Category III. The correct response, namely straight line motion.

SUBJECTS AND ADMINISTRATION OF QUESTIONNAIRE

Sixteen intact classes in two primary schools and one secondary school in Melbourne, Australia (N=357) participated in the study; similarly, nineteen intact classes in two primary schools and one secondary school in London, England (N=383) participated. There were two classes in each grade, except for ninth grade in England, where only one class participated. The numbers of boys and girls in each grade are given in Table 1. In Israel, 22 intact classes (N=628) participated in the study carried out by Eckstein and Shemesh (1993). The data of the Israel study are also given in Table 1.

Table 1

Numbers of subjects participating by year of study (grade) and country.

Year	Australia		England		Israel	
	Boys	Girls	Boys	Girls	Boys	Girls
2	19	20	8	6	28	23
3	27	20	19	24	31	18
4	28	22	28	18	29	28
5	25	20	21	24	19	32
6	32	26	23	16	22	22
7	27	21	29	20	35	37
8	-	-	23	21	36	33
9	16	19	9	9	32	29
10	13	22	31	18	28	37
11	-	-	22	14	28	27
12	-	-	-	-	22	32
Sum	187	170	213	170	310	318

The questionnaire was administered in the primary schools (Grades 2-6) by one of the researchers (S.G.E.). In the lower grades, wherever it was thought appropriate by the class's teacher, the questions were read aloud by the researcher while the children followed along. In the secondary schools, the test was administered by the class's teacher. It was explained to all the students that the purpose of the questionnaire was to improve methods of science teaching, and that their answers would not affect their grades. Question 3 required a written answer, which

was not considered appropriate for Grade 2, so results are not given on this question for Grade 2.

Each of the questionnaires was read by at least two of the three authors of this paper. Any initial differences (about 10%) were discussed, and the final categorization was accepted by both researchers.

RESULTS: SIMILARITIES AND DIFFERENCES ACROSS COUNTRIES

Our first research question concerns the differences between responses of students in each countries. To study this question, chi-square analyses were carried out to compare the responses in the three countries pairwise. The results are given in Tables 2 and 3 by question and by school grade. Very few responses in Australia and England were in Category I of Question 1 and in Category III of Question 3. Because there were so few responses in these categories, it was impossible to use all three categories of Questions 1 and 3 for the chi-square analysis. Therefore Categories I and II were lumped together for the analysis of Question 1, and Categories II and III were lumped together for the analysis of Question 3. Consequently, the number of degrees of freedom was df=1 for Questions 1 and 3; for the other two questions, df=2. For df=1, the critical values of chi-square at the 95% level is 3.84; for df=2, the critical value is 5.99.

Table 2

Values of chi-square for comparisons of children's reponses from different countries for Questions 1 and 2.

Question	Year	Countries being compared					
		England & Australia		Australia & Israel		England & Israel	
		Boys	Girls	Boys	Girls	Boys	Girls
1	2	0.34	0.09	0.48	11.93*	1.39	4.31*
	3	0.74	0.79	11.24*	2.69	5.53*	5.79*
	4	11.18*	0.46	9.73*	10.35*	0.09	6.81*
	5	3.07	1.60	8.03	1.19	1.38	6.44*
	6	0.81	4.11*	3.56	0.07	6.50*	2.94
	7	2.36	0.60	10.17*	3.86*	2.94	7.82*
	8	-	-	-	-	8.60*	10.37*
	9	0.26	5.45	0.37	12.54*	1.10	0.00
	10	0.91	0.24	4.47*	7.13*	2.72	9.20*
	11	-	-	-	-	1.30	0.01
2	2	0.12	0.11	1.26	5.16	1.22	3.47
	3	4.34	0.01	5.98	8.71*	0.19	9.45*
	4	2.25	15.15*	0.04	1.60	1.95	10.01
	5	3.30	2.77	0.34	1.32	2.77	2.85
	6	3.60	0.23	1.29	0.95	3.44	0.37
	7	4.18	0.74	2.04	2.24	4.67	0.28
	8	-	-	-	-	1.05	0.47
	9	0.04	0.22	1.88	2.70	1.46	0.62
	10	13.14*	2.44	6.55*	3.19	3.97	0.76
	11	-	-	-	-	0.28	1.89

*Significant difference at 95% level.

Table 3

Values of chi-square for comparisons of children's reponses from different countries for Questions 3 and 4.

Question	Year	England & Australia		Australia & Israel		England & Israel	
		Boys	Girls	Boys	Girls	Boys	Girls
3	3	6.76*	2.99	4.72*	4.08*	0.54	0.26
	4	25.82*	12.47*	8.99*	5.97*	5.93*	2.88
	5	0.15	0.70	0.72	0.15	0.20	0.26
	6	0.01	0.26	0.60	0.02	0.41	0.15
	7	0.21	1.45	0.07	0.39	0.60	3.70
	8	-	-	-	-	0.36	0.64
	9	3.14	4.55*	0.86	0.60	2.15	2.42
	10	1.97	2.17	0.09	0.39	2.40	1.13
	11	-	-	-	-	4.06*	3.64
4	2	1.71	3.06	7.77*	1.73	9.31*	6.84*
	3	2.34	0.19	0.48	1.81	4.13	0.99
	4	4.72	0.04	3.25	6.02*	6.60*	5.17
	5	0.82	5.75	2.43	3.82	1.39	9.99*
	6	2.12	3.67	8.26*	3.33	8.44*	6.80*
	7	1.65	0.04	0.45	2.46	1.69	3.00
	8	-	-	-	-	10.73*	2.08
	9	7.72*	2.63	1.72	0.05	4.73	3.13
	10	9.35*	0.43	15.19*	4.42	1.77	6.52*
	11	-	-	-	-	0.15	2.44

*Significant difference at 95% level.

We note that the chi-square values that compare Australia and England are generally very small. Of the 62 different comparisons, only 10 were significant at the 95% level. Four of the largest values of chi-square were in Grade 4, and two in Grade 10, suggesting that there may have been some special reasons for fluctuations in these grades. One possible reason may be copying among the students. A difficulty with our methodology was that the questionnaire was not administered under examination conditions: that is, the students sat quite close to each other, and there was probably a certain amount of copying. This could explain the large fluctuations in the results.

The similarity between the responses in England and Australia suggest that the data from the two countries can be grouped together for further analysis.

In contrast, the differences between Israel and England and also between Israel and Australia were large for two of the questions. This was particularly true for Question 1. Of the 36 chi-square values, over half (19) were greater than the critical value, implying significant differences.

For Question 2, the differences between Israel and the other two countries were not very large. This was also true for Question 3.

The differences between Israel and England were quite large for Question 4, with eight of the 20 values of chi-square showing significant differences. The differences between Israel and Australia were not as marked: only 4 of the 16 chi-values were greater than critical.

In accordance with the above results, we have lumped together the data for England and Australia for the purpose of displaying the percentages of students in each category,

and comparing these percentages with those of Israel. This is done for boys and girls separately in Figures 1 - 11. The figures also contain theoretical curves, so we postpone the discussion of the figures until after the theory is presented.

THE THEORETICAL MODEL OF STAGEWISE DEVELOPMENT

A mathematical model which describes stagewise development was formulated by Eckstein and Shemesh (1992a, 1992b) and applied to the development of Piagetian stages. It was also applied to the development of conceptions about motion among children in Israel (Eckstein and Shemesh, 1993), and fit the data very well. For the sake of completeness, we describe the model here. Further discussion is given in the above references.

Description of the model.

A set of N individuals, whose age is t_0 consists of a number of groups characterized by different stages of development, with n_i in the ith group. As time passes, individuals move from one group to another at a higher stage, so that n_i is a function of time. Our problem is to predict the values of the dependent variables n_i as functions of time t. The assumptions of the model are as follows:

Assumption 1. The groups are hierarchically ordered, so that people move from Group 1 to Group 2, but not vice versa, and not from Group 1 to Group 3.

Assumption 2. The number of people leaving a group in unit time is proportional to the number of people in the group--that is, the more people there are in a group, the more will leave it in unit time.

Consider Group 1. People leave this group as they mature, so that n_1 decreases. Assumption 2 says that the

rate of decrease, dn_1/dt, is proportional to n_1 itself. That is,

$$dn_1/dt = - \alpha n_1 \qquad (1)$$

The minus sign indicates a decrease in n_1 and a is a factor of proportionality.

Assumption 2 is the core of the model. The parameter a gives the rate at which transitions occur from Group 1 to Group 2. The most parsimonious assumption is this parameter is independent of n_1 and of age t, and this is therefore the assumption which should be investigated first.

Equations for Groups 2 and 3.

At this point, we assume for the sake of definiteness that there are only three groups, corresponding to three stages of development, and we find the differential equations for Groups 2 and 3.

The number n_2 of members of Group 2 change for two reasons: People move into the group, because those who leave Group 1 must enter Group 2 (by Assumption 1). Equation (1) tells us that this number is $+\alpha n_1$ per unit time. And people also move out of the group. According to Assumption 2, the rate at which they leave the group is proportional to n_2; i.e., it is equal to $-ßn_2$, where ß is a factor of proportionality. Taking both these effects into account, we get

$$dn_2/dt = + \alpha n_1 - ßn_2 \qquad (2)$$

where the second term on the right-hand side is the number leaving Group 2 per unit time. By the same reasoning, the rate of change of n_3 is the rate at which people enter from Group 2; i.e. it is equal to $+ßn_2$. Thus

$$dn_3/dt = + \beta n_2 \qquad (3)$$

This equation is not really necessary, because the total number in the group is a constant, N, so $n_3 = N - n_1 - n_2$.

Solutions of the Differential Equations

The solution of Equations (1) - (3) is straightforward. It is the following linear combinations of decreasing exponentials.

$$n_1 = A \exp(-\alpha t) \qquad (4)$$

$$n_2 = B \exp(-\beta t) - \alpha A \exp(-\alpha t)/(\alpha-\beta) \qquad (5)$$

$$n_3 = N - n_1 - n_2 \qquad (6)$$

Here A and B are constants to be determined by the initial conditions, i.e. the number in each group at the initial time t_0. These constants are therefore expected to change from sample to sample. This is not the case for α and β. These two constants determine the rate at which the populations of the groups are changing, and do not depend upon fortuitous initial conditions, that is, on the proportions in the sample chosen.

Characteristic Times (Half-lives).

The significance of the parameters α and β can be understood in another way by defining the characteristic time (half-lives), T_1 and T_2, as follows:

$$T_1 = \ln 2/\alpha; \qquad T_2 = \ln 2/\beta. \qquad (7)$$

The meaning of T_i is as follows: If there are n_i people in Group i at time $t = 0$, then at time $t = T_i$, only half the people, $n_i/2$, will remain in the group.

Application to cross-sectional studies

The model presented here is appropriate for a longitudinal study, where the groups are followed as a function of time. If we wish to apply it to cross-sectional studies, we must make a further assumption:

Assumption 3. The proportion of individuals in each group from a given population in a cross-sectional study is the *same* function of age as the proportion in a longitudinal study of the same population.

In a cross-sectional study, the dependent variables are not the number of individuals in each group, n_i, but rather the proportions, f_i, defined as n_i/N. Then

$$f_1 = a \exp(-\alpha t), \tag{8}$$

$$f_2 = b \exp(-\beta t) - \alpha a \exp(-\alpha t)/(\alpha-\beta), \tag{9}$$

$$f_3 = 1 - f_1 - f_2. \tag{10}$$

The functions f_i are thus functions of the four parameters in the theory, a, b, α, and β. To find these parameters, we first express the expected values of subjects in each group by using n_i(expected) = $N f_i$. Then

$$\chi^2 = \Sigma_i \frac{[n_i(observed) - Nf_i]^2}{Nf_i} \tag{11}$$

We see that chi-square is a function of the four parameters a, b, α, and β. We now can use a minimization program to find the values of these parameters which minimize chi-square. If this minimum value of chi-square is sufficiently small, then the theory is consistent with the data, and we have found the appropriate parameter values.

COMPARISONS BETWEEN COUNTRIES AND WITH THEORY

Question 1. Projectile Motion

As discussed above, the data for Australia and England (A&E) are lumped together for the purpose of displaying the data graphically. Figure 1 shows the percentages of boys

whose responses were in Category I as a function of grade. The line is the theoretical
curve obtained by the minimization process described above. It was fitted to the data from Israel. The data points for Israeli boys appear to fit the theoretical curve very well, and this is confirmed by the very low chi-square and high value of p=0.9 reported in Eckstein and Shemesh (1993). In contrast, the percentages of responses in Category I of the boys from A&E were all very low, although there are two apparently anomalous points at 4th and 9th grades, equal to

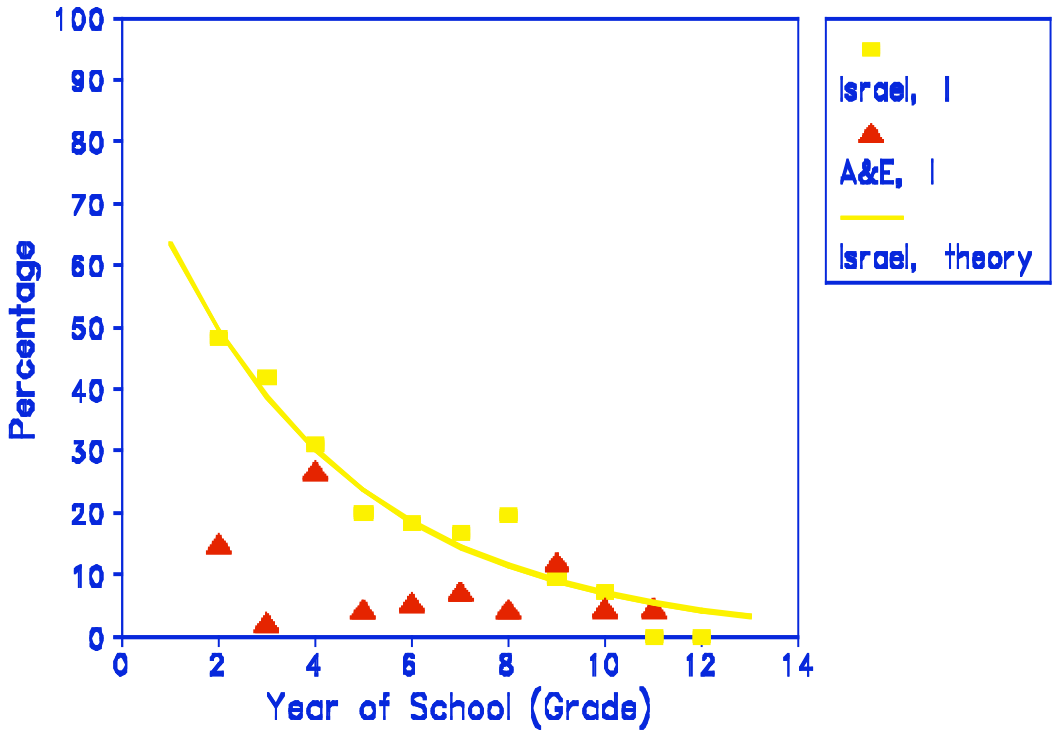

Figure 1. Question 1, projectile motion. Percentage of boys' responses in Category I. The curve is the theoretical fit to the data from Israel.

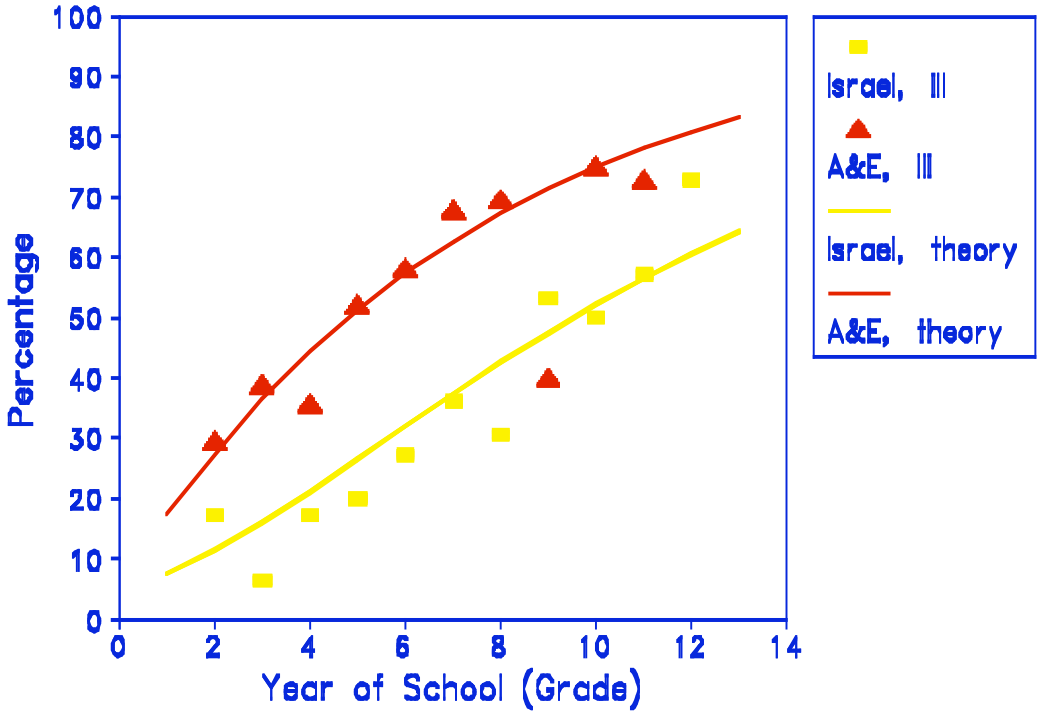

Figure 2. Question 1, projectile motion. Percentage of boys' responses in Category III. The curves are the theoretical fit to the data from Israel and A&E.

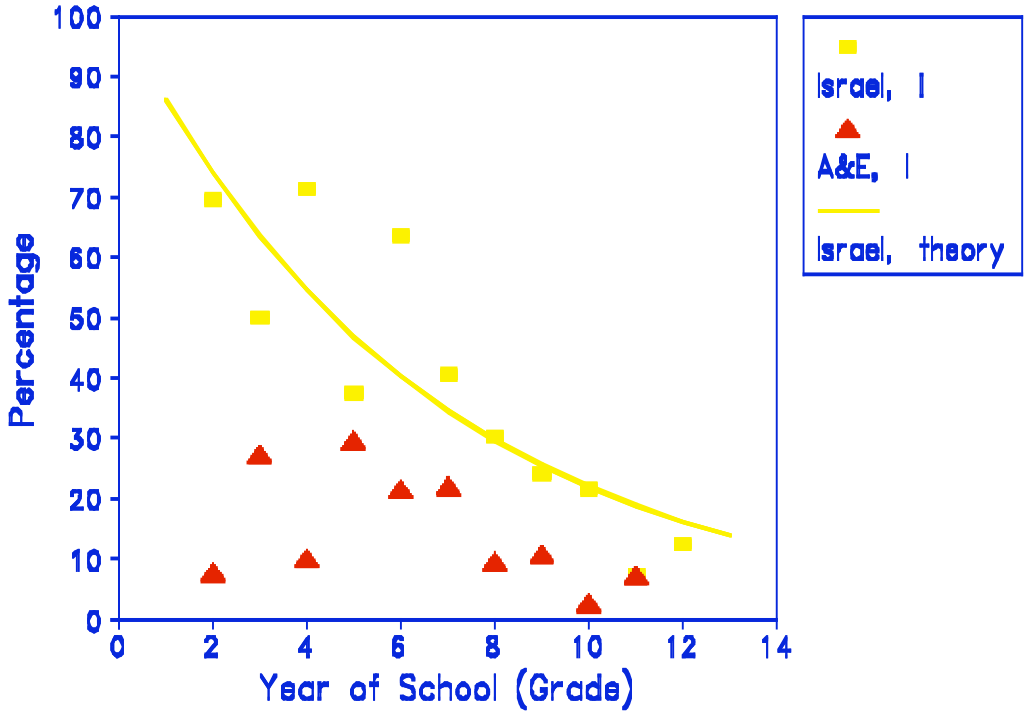

Figure 3. Question 1, projectile motion. Percentage of girls' responses in Category I. The curve is the theoretical fit to the data from Israel.

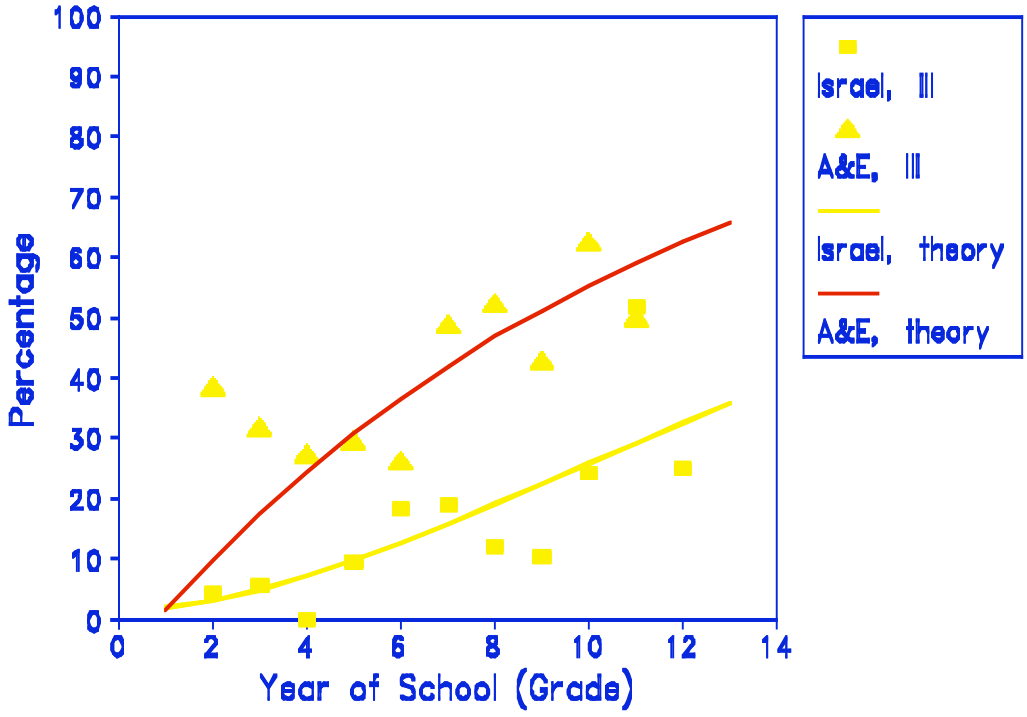

Figure 4. Question 1, projectile motion. Percentage of girls' responses in Category III. The curves are the theoretical fit to the data from A&E and Israel.

percentages for Israeli boys. The actual numbers were so low (in several grades there was only one boy in Category I!) that the usual 3-group chi-square analysis could not be done. However, when we lumped Categories I and II together, we could carry out a chi-square analysis for two groups (I & II and III). The data for Grade 9, which differs so drastically from the other data, was excluded from the analysis. The result is displayed in Figure 2 which gives the percentages of boys whose responses were in Category III. (It is not necessary to display the results for all three categories, because the sum of the percentages is 100%.) Here we see that the percentages of boys in Category III from A&E are

much higher than those of Israeli boys, except for Grades 4 and 9; and, except for these points, the theoretical lines fit the data well.

Figures 3 and 4 give corresponding data for girls. The fluctuations are much greater for the girls, but again we see that compared with Israeli girls, there are far fewer girls from A&E who responded in Category I, and far more who responded in Category III. The situation in Grades 2 and 3 appear to be anomalous, and these grades were excluded in the chi-square analysis.

In the two-group situation, appropriate for A&E, $a \approx 0$, and Equations 8 - 10 simplify to

$$f_1 \approx 0, \tag{12}$$

$$f_2 = b \exp(-\beta t), \tag{13}$$

$$f_3 = 1 - f_2. \tag{14}$$

Thus there are only two parameters in this case, b and ß (or alternatively, b and T_2). These parameters are given in Table 4, together with a comparison with the equivalent values for Israel. The errors for A&E were estimated by taking the standard error of the regression of $\ln(f_3)$ with t. Table 4 shows that the characteristic times, T_2, for movement into Category 3 are all very much greater in Israel than in A&E. To quantify this difference, we define the effect size (E.S.) as the <xfference between T_2 for A&E with T_2 for Israel divided by the standard deviation. The effect sizes are very large, 1.4 for boys and 2.2 for girls. Values for chi-square and p are also given. The fit of the data to the theory is excellent in all cases, except for Israeli girls, where it is marginal ($p = 0.051$).

Table 4

Parameters of the theory for Question 1, projectile motion. Comparison of Australia and England with Israel

Parameter	Boys		Girls	
	A&E	Israel	A&E	Israel
b	1.0±0.1	1.6±0.1	1.1±0.1	1.7±0.2
T_2 yr	5.2±0.5	6.3±0.5	7.9±1.8	10.7±2.1
E.S.*	2.2		1.4	
chi^2	2.99	12.3	5.35	28.8
df	7	18	6	18
p	0.89	0.83	0.50	0.051

*The effect size (E.S.) is defined as the difference in T_2 between groups divided by the standard deviation.

Question 2. Motion in Space

Figures 5 and 6 show the percentages of boys and girls responding to Question 2 in Categories I and III. The solid lines in the graphs are the theoretical curves fitted to the data from Israel. In contrast with Question 1, the data of A&E is very similar to that for Israel, with the data points generally falling along the theoretical curves. However, the data for Grades 2 and 3 are exceptions: there are far fewer responses in Category I than is the case for Israel. Boys' responses in Grade 9 were also anomalous: There were far more responses in Category I for boys in A&E than in Israel. It should also be noted that the fluctuations for A&E are much greater than for Israel.

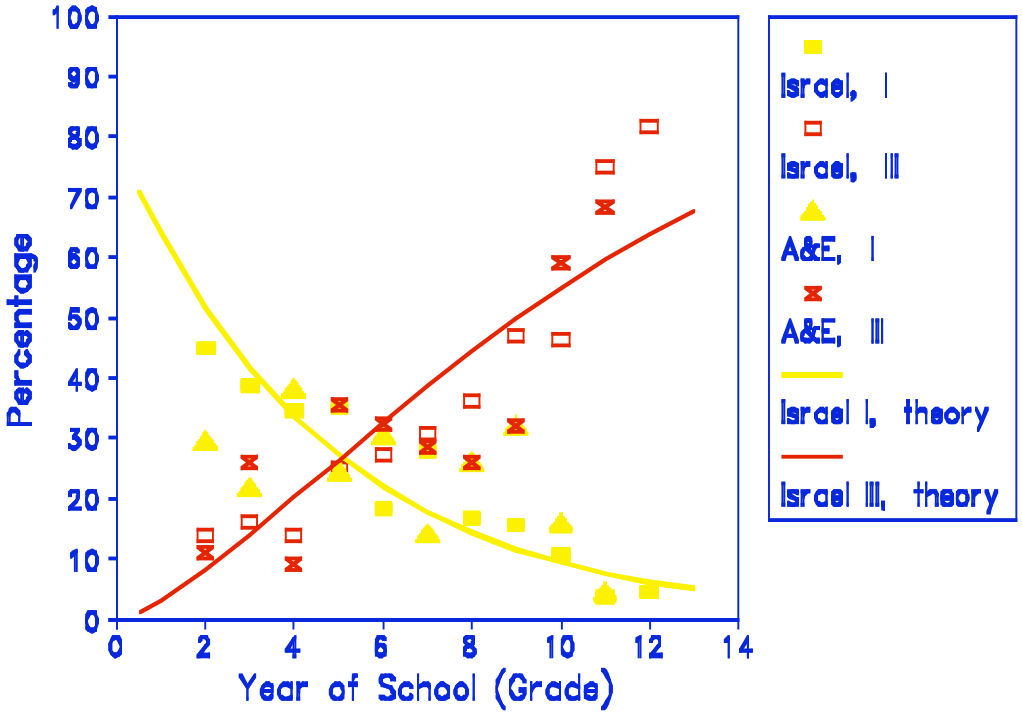

Figure 5. Question 2, motion in space. Percentage of boys' responses in Categories I and III. The curves are the theoretical fit to the data from Israel.

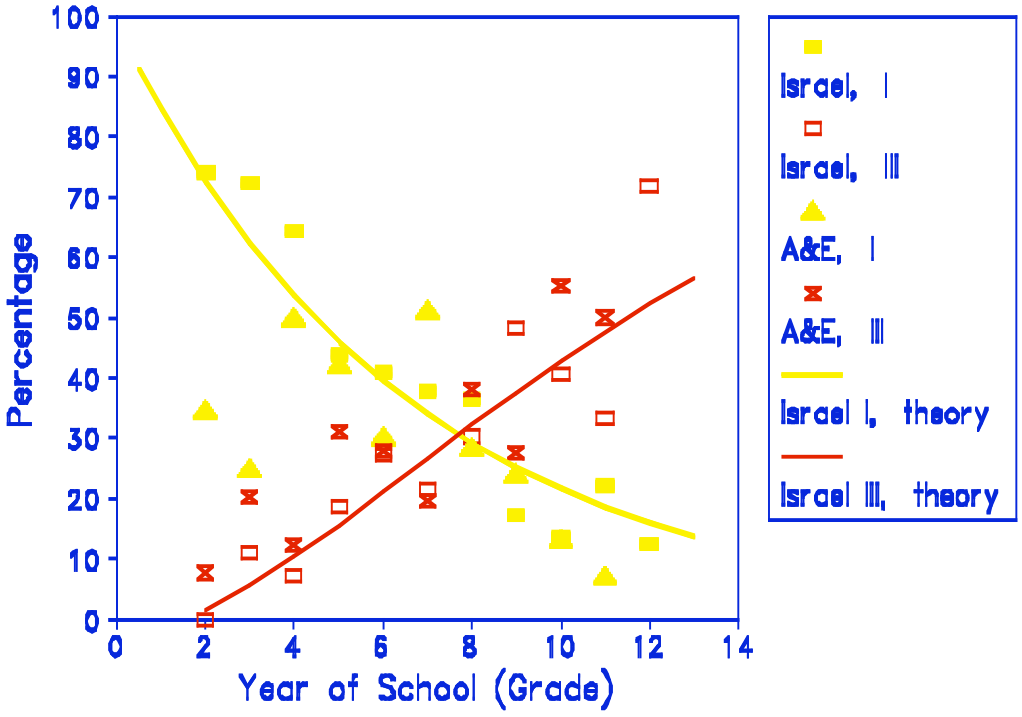

Figure 6. Question 2, motion in space. Percentage of girls' responses in Categories I and III. The curves are the theoretical fit to the data from Israel.

Even though there were large fluctuations in the data, a chi-square analysis was carried out to compare the results with the theoretical model. Because of the anomalies, Grades 2 and 3 were excluded from the analysis as well as Grade 9 for boys. The results are summarized in Table 5. The errors were estimated by a procedure discussed by Eckstein and Shemesh (1993).

The results of this analysis were quite surprising. It was found that the characteristic times T_1 and T_2 for both boys and girls in Australia and England are the *same* as the characteristic times in Israel, within experimental error. The effect sizes for boys are less that 0.5, and for girls they are less than 0.1. This is especially surprising when taken in conjunction with the results for Question 1, which showed very large differences between the two groups.

Table 5

Parameters of the theory for Question 2, motion in space. Comparison of Australia and England with Israel

Parameter	Boys		Girls	
	A&E	Israel	A&E	Israel
a	0.76±0.43	0.79±0.23	1.00±0.58	0.98±0.20
b	2.67±0.80	2.38±0.28	15.0±6.5	13.7±0.28
T_1 yr	3.71±0.95	3.25±0.40	4.62±0.62	4.56±0.44
E.S.		0.48		0.10
T_2 yr	5.42±1.03	5.06±0.35	4.95±0.69	4.92±0.09
E.S.		0.35		0.04
chi^2	19.1	14.6	18.7	18.0
df	10	18	12	18
p	0.04	0.69	0.10	0.46

In this connection, it should be noted that the differences in Question 1 favored A&E, whereas for Question 2, all characteristic times in Israel were shorter (although not significantly so) than the corresponding times in A&E, so that the results were slightly *better* in Israel.

The fit to the theory was excellent for Israeli boys ($p=0.69$) and girls ($p=0.46$), acceptable for girls from A&E ($p=0.10$) and marginal for boys from A&E ($p=0.04$). The striking resemblance of the data between the groups, and their qualitative agreement with the theory makes it plausible that replications of the measurements with better statistics will give better agreement with theory.

Question 3. Relative motion.

Figures 7 and 8 show the percentages of boys' responses to Question 3 on relative motion in Categories I and III respectively. The heavy line in Figure 7 is the theoretical curve fitted to the data of Israel, and the light line is the theoretical curve fitted to the data from A&E. The solid line in Figure 8 is the theoretical line fitted to the data from Israel. There were too few responses of boys from A&E to allow for a chi-square analysis with three groups, and consequently there is no theoretical line for A&E in Figure 8.

The percentages of responses of girls in Categories I and III are shown in Figure 9. The number of responses for Israeli girls in Category III were too few for a chi-square analysis, as was the case for both boys and girls from A&E. Therefore we have theoretical curves only for Category I. The heavy line is fitted to the data for Category I from Israel, and the light line is fitted to the data from A&E.

For the situation in which there are only two groups, I and II & III combined, we find only two theoretical parameters, a and α, or equivalently, a and

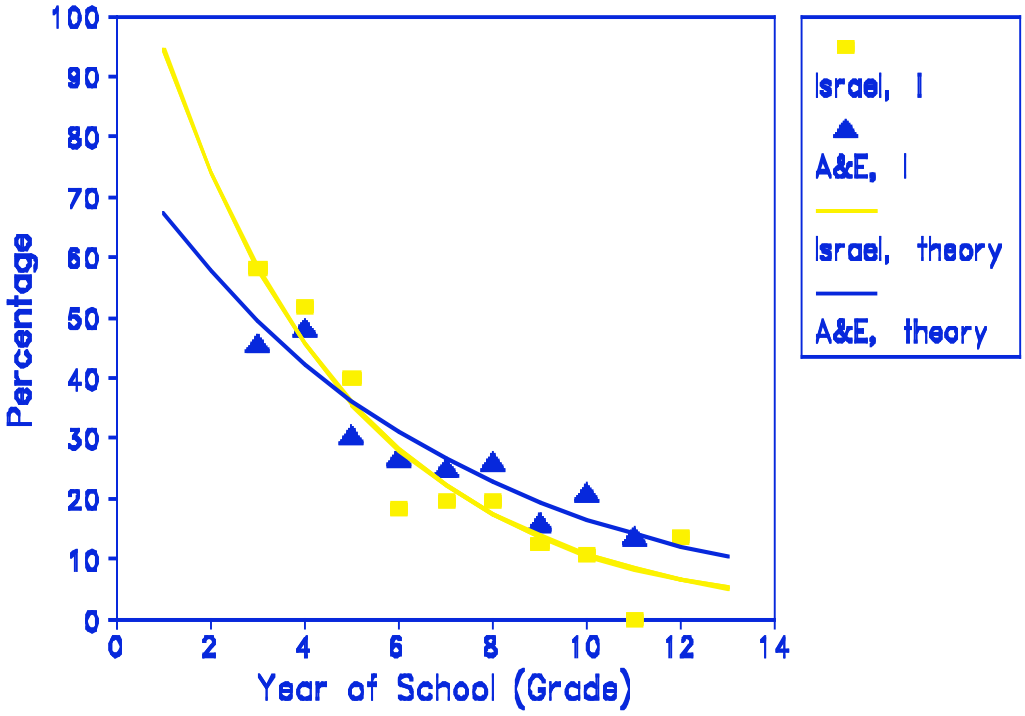

Figure 7. Question 3, relative motion. Percentage of boys' responses in Category I. The curves are the theoretical fit to the data from A&E and Israel.

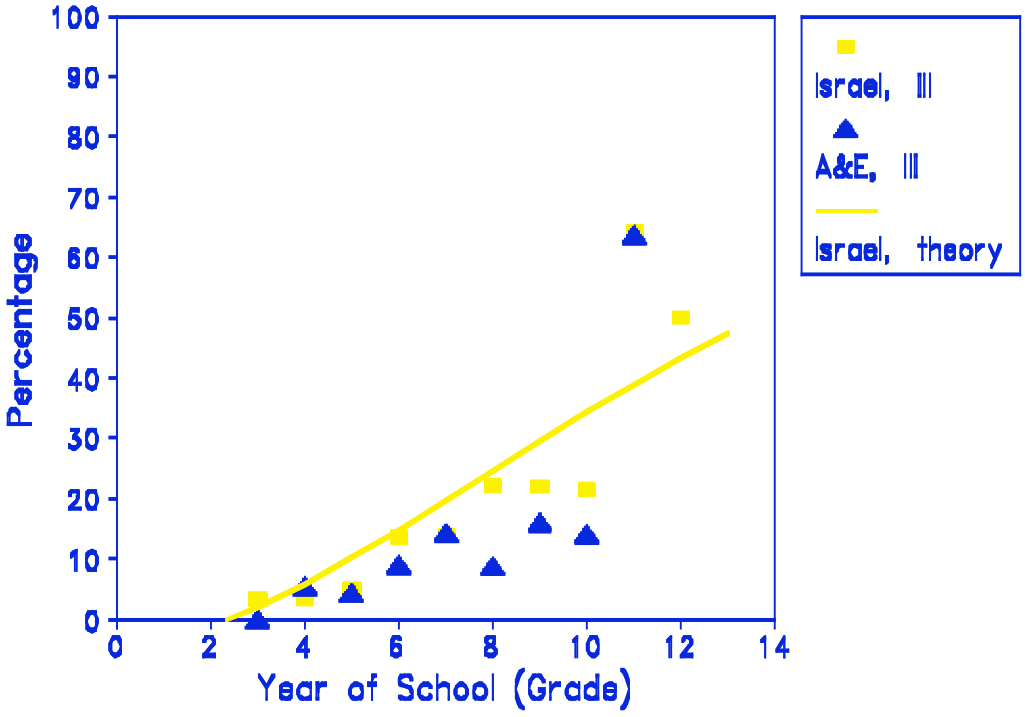

Figure 8. Question 3, relative motion. Percentage of boys' responses in Category III. The curve is a theoretical fit to the data from Israel.

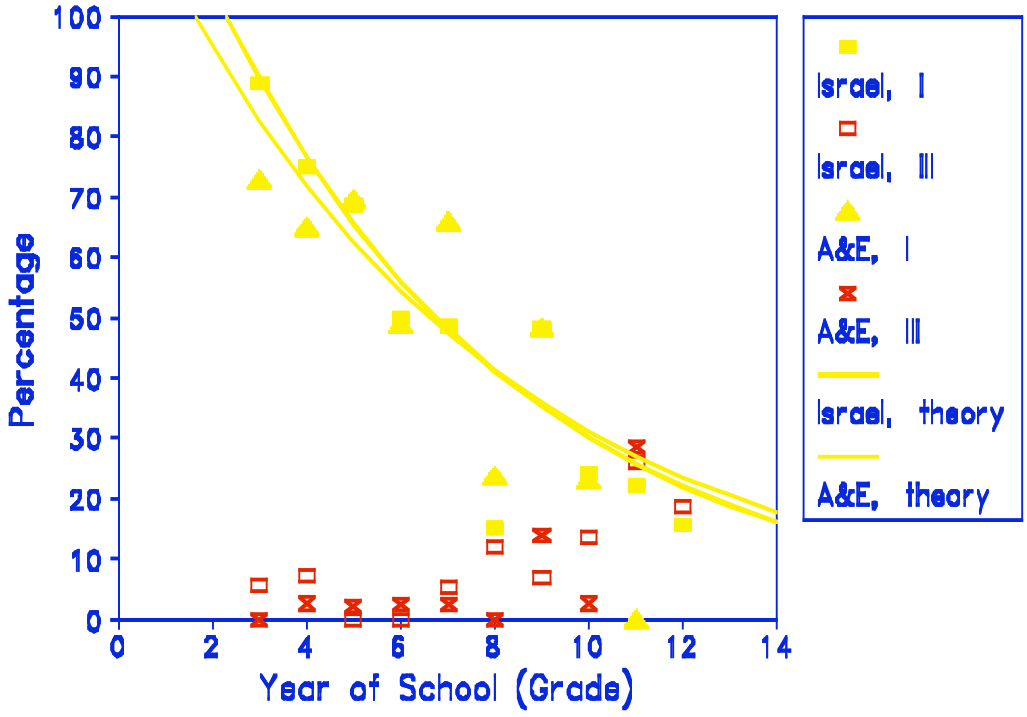

Figure 9. Question 3, relative motion. Percentage of girls' responses in Categories I and III. The curves are the theoretical fit to the data from A&E and Israel.

Table 6

Parameters of the theory for Question 3, relative motion. Comparison of Australia and England with Israel

Para-meter	Boys		Girls	
	A&E	Israel	A&E	Israel
a	0.74±0.11	1.20±0.24	1.10±0.40	1.43±0.48
T_1 yr	4.71±0.70	2.86±0.52	5.65±1.55	4.45±0.83
E.S.*	3.1		1.1	
chi^2	3.04	18.7	20.1	10.6
df	7	16	7	8
p	0.88	0.29	0.005	0.22

T_1. The theoretical parameters for A&E are given in Table 6, and compared with those of Israel. From the table we see that in three of the four cases, the fit of the theory to the data was excellent with p values of 0.22, 0.29 and 0.88; in the fourth case (girls from A&E) the data did not fit (p=0.005)

The surprising result is the large differences in the parameters, favoring the Israeli children. The effect size was particularly large for boys (3.1). For girls the effect size had the large value of 1.1, but since the data is not fitted by the theory, we should not take this value too seriously.

A short characteristic time, T_i, means that it takes a short time for children to leave the ith stage, and hence the children develop more rapidly. In Question 1, children from A&E had the shorter characteristic times; in Question 2 the characteristic times were very similar for both groups; while in Question 3, Israeli children had the shorter characteristic times. These results are very puzzling because we would intuitively suppose that there would be an hierarchical ordering, in which groups with higher ability would exhibit their ability consistently, at least within a small subject range. In this research, the subject range is very small indeed, with all questions dealing with different aspects of motion. Yet the results show that there are large differences between groups (and presumably between abilities) but no possibility of hierarchical ordering.

Question 4. Circular Motion

Figure 10 gives the percentages of boys' responses to Question 4, circular motion, in Categories II and III. The percentages for girls are given in Figure 11. These results do not show any evidence of development, and it is impossible to fit the mathematical theory to the data. A possible reason

for this may be that responses in Category II imply a belief in the misconception of circular impetus; and this misconception, while being incorrect, is probably requires just as much sophistication as the correct response. We note that in this case there seem to be more responses in Category 3 among the Israeli children than among the Australian or English children.

Table 7 gives a summary of the overall percentages of responses in each category by country. The percentages giving correct answers in Australia and England are similar to the 47% reported for university students by McCloskey, *et al.* (1980); but the responses in Category II are about half the 51% reported by the above authors. The responses of the Israeli children was somewhat different: about 60% gave the correct response.

This question has special theoretical significance, even though (or perhaps, *because*) the data cannot be fit by the theory. A theory can be said to be scientific only if it is possible to falsify it. The question is, are there any data sets which *cannot* be fitted by the theory? If there aren't any, then the theory cannot be falsified. Here we have a clear example of a data set which cannot be fit by the theory, and hence the theory is indeed falsifiable. This gives added significance to the other cases in which the theory fits the data well.

Table 7

Question 4. Circular motion. Overall percentages in each Category by country, with standard deviations in parentheses.

Cate-gory	Boys			Girls		
	Aus.	Eng.	Israel	Aus.	Eng.	Israel
I	26(3)	29(3)	15(2)	33(3)	36(3)	15(2)
II	26(3)	23(3)	24(2)	28(3)	20(3)	28(3)
III	48(4)	49(3)	61(3)	39(4)	43(3)	58(3)

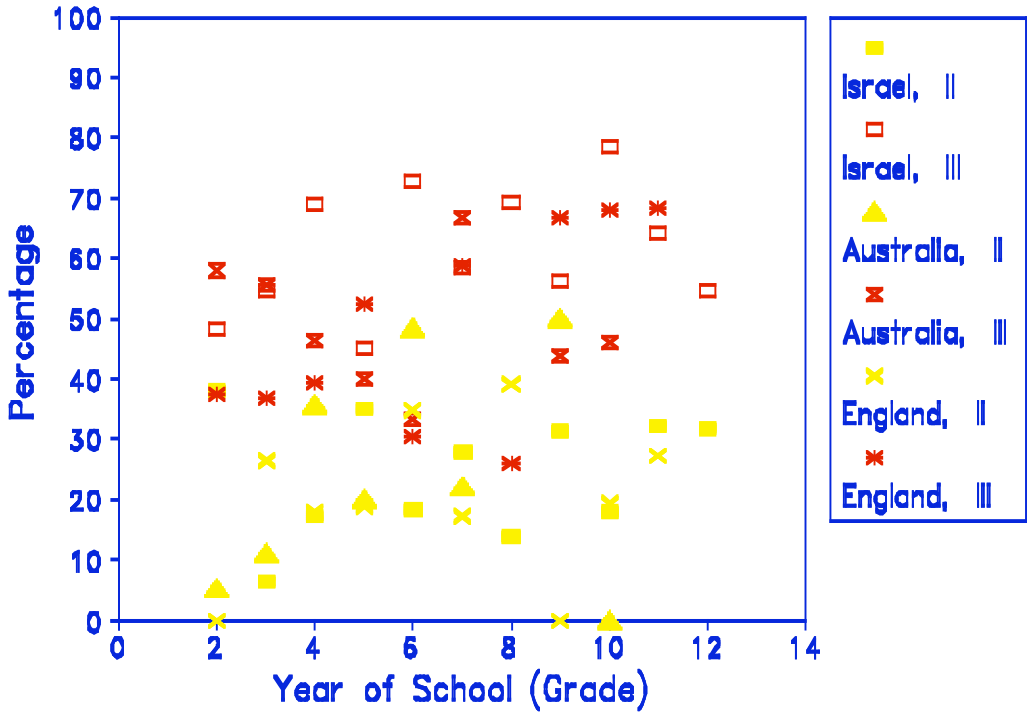

Figure 10. Question 4, circular motion. Percentage of boys' responses in Categories II and III.

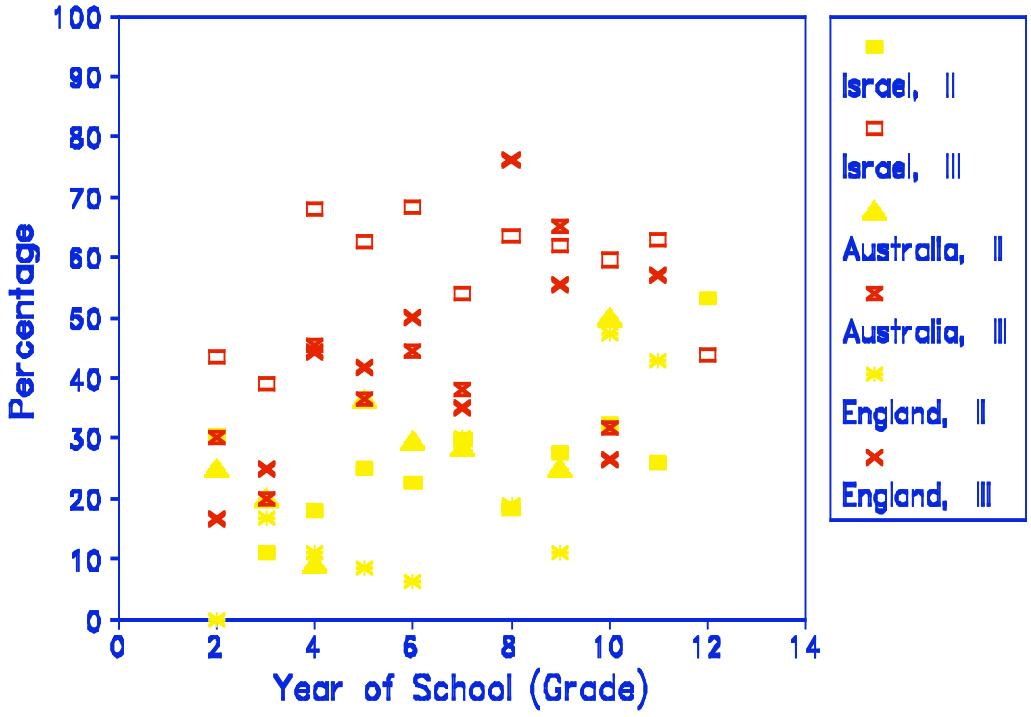

Figure 11. Question 4, circular motion. Percentage of girls' responses in Categories II and III.

DISCUSSION

Our first research question was whether the same stages were apparent in the development of the concepts of motion in England and Australia as in Israel. This answer to this was clearly positive: the same categories of answers were given by children in all three countries, and the progression of responses as a function of age was very similar.

Our second research question concerned the detailed similarities and differences between development of concepts of motion in the three countries. A question by question and year by year comparison of the frequencies of responses in the three countries showed that the development of

concepts of motion in Australia and England were very similar. However, there were substantial differences between Australia and England on the one hand, and Israel on the other hand. These differences were quite puzzling, because for one aspect of motion--projectile motion--the responses in Australia and England were much better than those in Israel; for two other aspects--relative motion and circular motion--the responses in Israel were better than those in Australia and England; and for yet a fourth aspect--motion in space--the differences between the groups were negligible.

Three of the questions dealt with aspects of motion about which students develop their conceptions by their own personal experience, but one question dealt with knowledge that the students could have acquired only through the media (and possibly, but not probably, in school). On this question only--motion in space--the responses of the children in the different countries was indistinguishable. Could this be additional evidence for the `global village'?

Our final research question concerned the application of the theoretical model of stagewise development of Eckstein and Shemesh (1993). There were three questions which showed stagewise development, and so there were 12 cases in which it was possible to compare the theory with the data: for each question, for boys and girls separately; and for each group, A&E and Israel. Of these 12 cases, the theory fit 8 cases very well, with values of p between 0.22 and 0.89; in one case the fit had the acceptable value of $p = 0.10$; two cases were marginal, with $p = 0.05$ and 0.04; and one case did not fit at all ($p = 0.005$). These results suggest that there is some validity to the theory, and that it would be useful to continue its investigation.

We have already noted that the fluctuations of the data in Australia and England were very large. We expect that

improved statistics would remove some of the fluctuations. We also speculate that the fluctuations may have been caused by student copying which may have been widespread; further studies of the theory should attempt to eliminate this problem. We expect that research with improved methodology will provide a better test of the theory.

The power of the mathematical tool is manifested by its ability to characterize large amounts of data with a small number of parameters. To the extent that the theory is valid, four parameters are enough to describe changes in a group as it develops in time through three different stages, and two parameters are enough for two different stages. As we have seen, these parameters are very useful in comparing different groups. If a characteristic time for one group is shorter than the corresponding characteristic time of another group, then we know that the first group is developing more rapidly. We believe that use of mathematical models, such as the one in this paper, will be an important aid in helping us to understand children's cognitive development.

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