Third Misconceptions Seminar Proceedings (1993)

Paper Title: CONCEPT-RELATIONSHIP KNOWLEDGE STRUCTURES: APPLICATIONS IN EDUCATION

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Keywords: Knowledge representation, Concept-names, Relations of arities, Syllabuses, Design, Analysis, Presentation, Analogical reasoning (formal), Theory of teaching/learning
General School Subject:
Specific School Subject:
Students:

Macintosh File Name: Geldenhuys - Education
Release Date: 6-15-1994 G, 11-8-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

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CONCEPT-RELATIONSHIP KNOWLEDGE STRUCTURES: APPLICATIONS IN EDUCATION

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Paper to be read by prof H Helm

at the Third International Seminar:

Misconceptions and Educational Strategies

in Science and Mathematics

Cornell University

August 1-4, 1993

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KEYWORDS

Knowledge representation
Concept-names
Relations of arities ≥ 2
Syllabuses: Design
Analysis
Presentation
Analogical reasoning (formal)
Theory of teaching/learning

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INTRODUCTION

Some years ago Hestenes suggested ways to approach a science of teaching [Hestenes 1979]. Here we informally describe one possible means to a science of education. A formal model is introduced [Geldenhuys, Helm & Van Rooyen 1992], in terms of which one can, among other things, make predictions about the effectiveness of teaching/learning. These predictions can be tested and the results of these tests can be used as feedback to modify the model.

In this paper we will use the term curriculum to denote a collection of (hopefully interrelated) courses that make up some qualification such as a diploma or degree. The term syllabus will indicate the detail of the relationships and concepts that constitute a course. The model that we describe partially and informally here applies to the design, analysis and presentation of both curricula and syllabuses [Geldenhuys & Van Rooyen 1993].

At the semantic level our model is a linguistic one. Its construction starts with a list of statements of relationship, for example

i: Two <u>equal</u> <u>sets</u> have precisely the same <u>members</u>

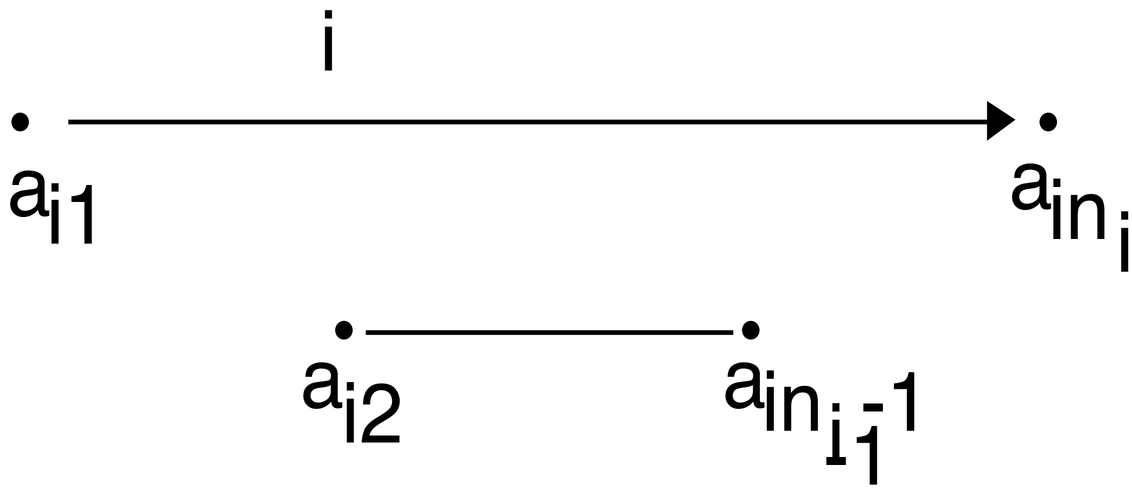
in which the marked words, called concept-names, are the terms that are to be taught/learned. Here i is a unique statement code name, often a number. The semantics of the model thus consists of a list of statements constructed in such a way that every concept-name is related to at least one other concept-name, i.e. the corresponding relations are of arities ≥ 2. From this list we construct the syntax of the model in a form that is a generalized and formalized extention of the notion of a concept map [Novak & Gowin 1984, Novak 1990a, Novak 1990b].

The syntax of the model arises by entering i and the sequence of concept-names in statement i, in the order in which they appear in statement i, for each i. Notice that a concept-name may be duplicated several times in a given statement. Each occurrence of a concept-name must be entered with each i. For each i then, we get a tuple of concept-names in the form

$$i;\ (a_{i1},\ a_{i2},\ldots\ldots,a_{in(i)})$$

for some $n_i \geq 2$. These tuples may be entered into a computer, each with the associated i, and constitute the syntax with which we work formally.

To visualize the model we plot a graph-like diagram in which there is a one-to-one correspondence between the set of vertices and the set of concept-names. For each tuple i, we draw an arrow from a_{i1} to $a_{in(i)}$ and label it with i. Notice that there is at most one arrow between any two vertices. It follows that an arrow may have more than one i-label. For each relationship i we then have an arrow



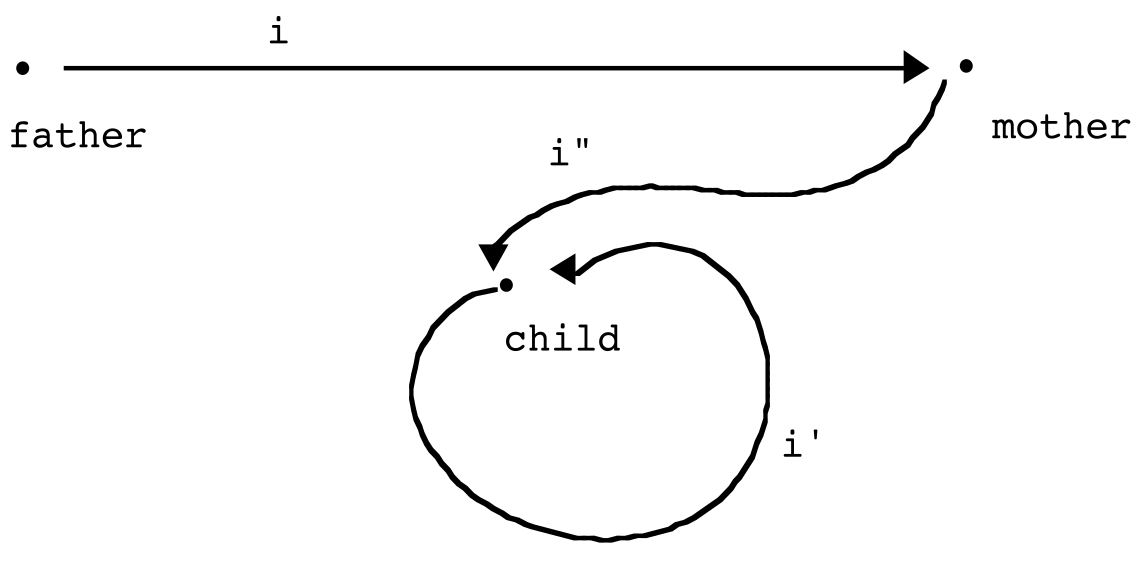
in the diagram. It is clear that there may be a linking of arrows producing paths in the diagram. Such paths reveal more complex relationships among the concept-names. In general we may say that the more such paths the richer our model. It is possible to force meaningful paths by permuting the concept-names in any tuple i. This involves re-stating the relationship i in various ways. As an example, consider the statements

i: The father of this child is married to this child's mother.

i': This child's father is married to the mother of this child.

i": The mother of this child is married to the father of this child.

and so on, all of which represent the same relationship.
In this case the diagramatic representation of this syntax is



A more complete form of the diagram can be drawn. In it we enter the middle part of the tuple i onto the appropriate arrow. Thus our first diagram has arrow label $i; <a_{i2},......,a_{in(i)-1}>$. This notation helps to emphasize the complexity of the paths and the fact that the structure is "algebraically closed".

THE MODEL

We call our model a Concept-Relationship Knowledge Structure, abbreviated to CRKS. It is a formal mathematical structure about which we have proved some interesting theorems that have powerful practical applications when the structure is used to represent a syllabus in a theory of teaching and learning. To arrive at our model we put some practical constraints on the general relation structures with which the work began [Van Rooyen 1976, Van Rooyen et al 1981, Van Rooyen 1990]. We will express these constraints in terms of a diagram of an arbitrary CRKS.

> (i)The vertices in the diagram of a CRKS are in one-to-one correspondence with the set of concept-names of that CRKS. Every i-label on an arrow represents a statement of relationship, corresponding to a relation of arity ≥ 2, among concept-names. Thus every vertex (concept-name) in a CRKS is related to at least one other vertex.

The implications of (i) are the following: A concept-name does not exist unless it is related to at least one other concept-name, so concept-name and relationship are taken to be primitives of our theory, i.e. these two terms can only be established, in conjunction, by means of examples. Next, if we delete a vertex from the diagram of a CRKS then every tuple which involves that vertex in any way disappears from the diagram since no relationship in the name of that vertex (concept-name) can now be written down. This is a general effect called strong vertex vulnerability, and in the case of a CRKS it expresses context sensitivity. The collection of those tuples that involve a given vertex v, all of which are deleted when v is deleted, constitutes a substructure called the context schema of concept-name v with respect to the whole CRKS. It defines the meaning of concept v relative to that CRKS. As we add more relationships that mention v to the CRKS we change the meaning of concept v relative to the resulting CRKS. Thus "concept-name v" refers to the vertex v alone while "concept v" refers to the current context schema of v. Various gauges can be used to judge how well-integrated a concept-name is in a given CRKS [Geldenhuys, Helm & Van Rooyen 1992]. Weakly integrated concept-names can be flagged using these gauges, and the designer(s) of a syllabus-CRKS may then add in more relationships that involve these vertices (concept-names).

> (ii)There must be no circuits in a CRKS. Essentially this condition avoids situations in which a concept-name is described in terms of itself in a circular fashion, although our model does allow spiralling, in which a preliminary meaning of a concept-name is used to revise and extend the relative meaning of that concept-name.
>
> (iii)There must be at least one vertex with no incoming arrows. Such vertices represent primary concept-names, called primaries.
>
> (iv)There must be at least one vertex with no outgoing arrows. Such vertices represent goal concept-names, called goals. The intention is that the primaries are known, from previous learning, and the goals are the concept-names toward which teaching/learning is directed.
>
> (v)A CRKS must be complete, i.e. every vertex must have at least one arrow to or from it, or both.
>
> (vi)Every vertex of a CRKS must be derivable from the primaries of that CRKS. This means the following: The primaries are trivially derivable by means of a path of length zero, i.e. they are "known" from previous learning, and for each non-primary vertex v there must be at least one path from some primary to v. Futher, every such path must have the property that every vertex in every tuple involved in that path is

either primary or is derivable from the primaries. The interpretation of this condition is that every vertex v in a CRKS is either primary, i.e. already learned, or is learnable in terms of relationships with already learned primary and non-primary concept-names in that CRKS. Every relationship in a CRKS is treated as a rule of inference, so this condition is analogous with the definition of a formal deduction in first order logic.

Given any general relationship structure we have an automated test that will tell us whether or not it is a CRKS. This involves a search technique which we call a limited access cascade in our theory. If we feed the limited access cascade algorithm the primaries it will find only those vertices which can be reached by means of paths that use only vertices already found. If this search generates precisely the whole structure then that structure is a CRKS; if not then we do not have a CRKS. We thus have an automated technique for checking the integrity of a syllabus structure at each stage of its development. In practise it turns out to be relatively easy to construct a syllabus-CRKS. A team of subject experts can read relationships into the structure, together or as independently acting individuals, and at each stage one can check for weakly integrated concept-names and for CRKS form. Sub-CRKS's are easy to define one at a time, and we can join up any chosen collection of CRKS's and then check that the resulting combination is a CRKS [Geldenhuys, Helm & Van Rooyen 1992]. This is advantageous in the sense that we can design small sections of a syllabus at a time and will never have to deal with huge amounts of information in single clumps. Experience indicates that about 100 statements may be the maximum that can conveniently be coped with in one sub-CRKS.

We should note that the words not marked in the statements are those that we assume to have known meaning, preferably as established in previous CRKS's. We do not want to deal with these words as concept-names in the current CRKS, i.e. the current CRKS says nothing explicit about unmarked words. It is also important to note that the CRKS model itself generally forces redundancy: Certain relationships must often be restated in different ways in order to achieve CRKS form. This is not a problem: For example, the statement

<u>speed</u> is <u>equal</u> to <u>distance</u> covered divided by <u>time</u> taken

should be re-written in other ways such as

<u>distance</u> covered is <u>equal</u> to <u>speed</u> multiplied by <u>time</u> taken etc,

to help the learner to fully understand the relationship between speed, equal, distance and time.

Our model also implies some presentation strategies for a syllabus. Several of these are induced naturally by the nature of the model, each essentially describing a hierarchy of nested sub-CRKS's that cover the whole CRKS in learnable steps. Some strategies deal with ordering paths and some with ordering concept-names, but all arise directly from the notion of derivability in a CRKS. Application of a version of Menger's theorem to a CRKS enables us to find "quasi-minimal" sets of paths with the property that every member of such a set must be taught/learned. Application of "tuples bases" for a CRKS will yield minimal sets of tuples with the property that every tuple in such a set must be taught/learned. Using the hypergraph associated with a CRKS presents us with another way of determining such minimal sets of tuples. (Notice that we distinguish between presentation strategies and interaction modes, where by the latter term we mean blackboard and chalk, laboratory exercises, CAI lessons and so on.)

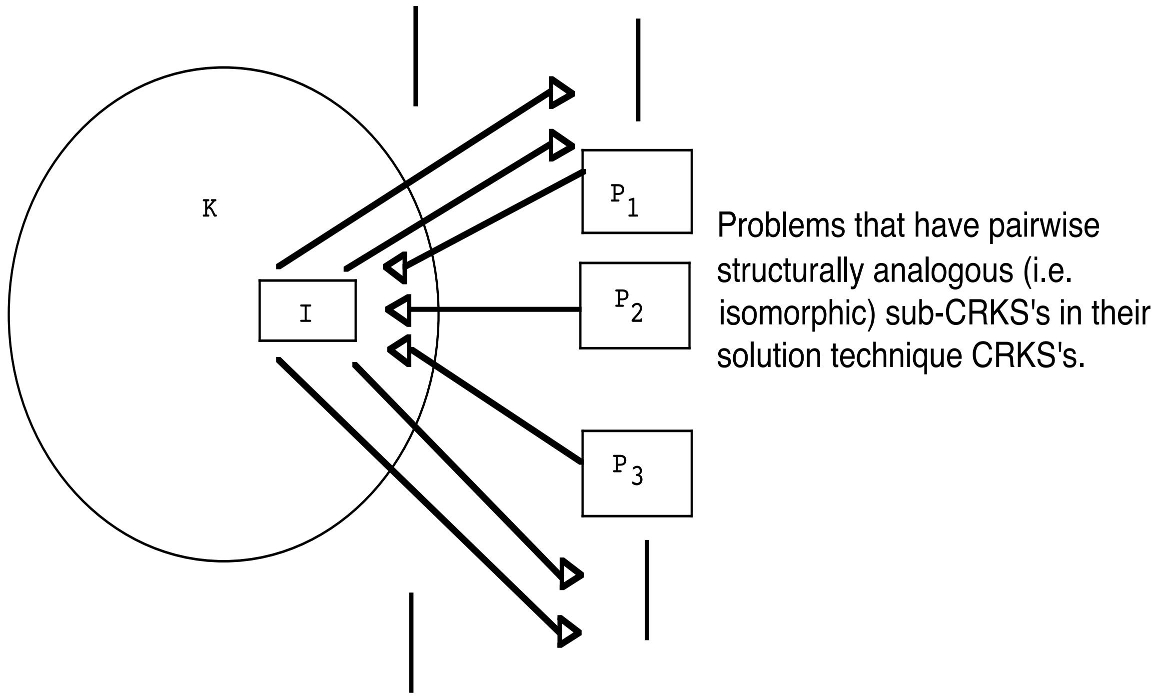
STRUCTURAL ANALOGY.

In our theory we formally define an isomorphism between [sub-]CRKS's. An isomorphism identifies concept-names in one sub-CRKS with concept-names in the other in such a way that tuples are preserved. The two isomorphic sub-CRKS's are said to be structurally analogous. Semantics, in the form of statements of relationship, is not involved as this kind of analogy isolates corresponding syntactical structures. However, semantics may be used as a heuristic device in trying to establish such analogies. Leaving out the semantics enables one to automate the search for analogies.

Suppose that we have a known CRKS K and we are investigating a new phenomenon. In this new situation we establish concept-names, and relationships among them, to begin describing that situation. Suppose that this new knowledge constitutes a CRKS N. We have an algorithm, based on an idea by Stetter [Stetter 1992], that allows one to make an automated search for a sub-CRKS K' of K that is structurally analogous with N. If any such K' can be found, and there may be several of them or none, we reverse the mapping. We then choose tuples in K that share at least one vertex with K' and map them across to become hypothetical predicted tuples that are attached to N. We test these predicted tuples (the syntax) by trying to fit consistent concept-names and relationships to them in the new situation i.e. by trying to provide semantics for the predicted tuples. If this test fails we temporarily ignore the relevant prediction and try another. Successful predictions extend the range of the mapping, each new prediction sharing as many vertices as possible with the current range. At each stage the range constitutes a new CRKS that includes the previous one. A measure of the success of a particular structural analogy is the number of successful predictions one can make on the basis of the known, familiar CRKS K. The final sub-CRKS of K is called a model of the new situation. Several different models can be used for parts of the new situation with various degrees of success, even if they contradict each other on some predictions. Empirical test is the measure that will eliminate certain models, and semantic connections can also play a role in the choice between various models of the same new situation. Further, one model can often be used for different new situations. The richer K is, and the more useful models found in K, the easier it becomes to use modelling (i.e. reasoning by analogy).

Another use of CRKS isomorphisms is connected with problem solving. We have shown that the top-down specification of a problem solution can easily be converted into a CRKS that displays the solution technique for that problem. Suppose, for example, that we have problems P_1, P_2 and P_3 that have pairwise structurally analogous, (i.e. isomorphic), sub-CRKS's in their solution technique CRKS's.

We can then abstract these sub-CRKS's and join this abstracted (partial) solution technique, in the form of a CRKS I, to our known CRKS K. Thus we have



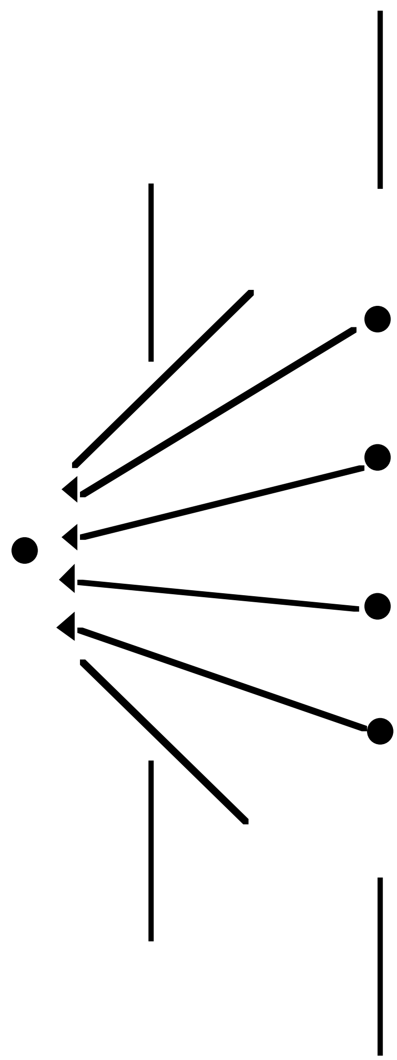
The arrows directed to the left represent isomorphisms that abstract the (already discovered) structurally analogous sub-CRKS's of the solution technique CRKS's of P_1, P_2 and P_3. These abstraction isomorphisms constitute structural analogies that isolate the invariant CRKS I which is joined to K, expressing the fact that the solution technique CRKS's of P_1, P_2 and P_3 have pairwise isomorphic (structurally analogous) sub-CRKS's.

The arrows directed to the right indicate attempted isomorphisms, which we call algorithmic structural analogies, that specify how to try to solve new problems using the solution technique CRKS I that was learned while solving P_1, P_2 and P_3. The definition of such an isomorphism inherently contains the algorithm for the solution of problems that are at least partially similar to P_1, P_2 and P_3.

Just as the CRKS model can be of general use for design, analysis and presentation of a syllabus, this CRKS view of problem solving expressed in terms of structural analogy, and indeed the whole spectrum of uses of structural analogy, can be important for teaching and learning. For a more complete treatment of structural analogy see Geldenhuys, Helm & Van Rooyen [1992]. Our (limited) experience with CRKS's suggests that much of human reasoning and learning may be based on structural analogy. Pointing out structural analogies to students seems to make the acquistion "new" knowledge easier for them.

PHASES OF LEARNING IN THE CRKS VIEW

The situation shown in the following diagram, which involves a number of vertices on the right that share some common invariant that is associated with one vertex on the left, occurs at five levels.



The relationship represented by the arrows, which has a large (and possible growing) domain and a single element codomain, is called a primitive binary relationship. The situation constitutes the most primitive kind of CRKS. At the prelinguistic level of human development the vertices on the right represent some perceptions that are noticed because they have something in common. At this stage this invariant does not yet have a name, so we call the vertex on the left an attention vertex: It merely indicates that something has attracted the young learner's attention. The linguistic stage starts, in the CRKS-model, when the attention vertex is assigned a primitive concept-name.

By a primitive concept-name we mean some symbol that indicates what is common to a number of observations as represented by the vertices on the right. A primitive concept-name thus arises from what is common (invariant) among a number of examples. Thus one might establish the primitive concept-name "red" for a learner by means of examples such as red ball, red car, red roof, et cetera. The concept-name "red" is abstracted, the other words just being "noise" at that stage. Secondary concept-names are those that are established solely by means of relationships with other concept-names. [See also Thelen 1986, Okebukola 1990, West & Fensham 1974, Beeson 1981, Otero & Campanario 1990, Heinze-Fry & Novak 1990, Summers 1982, Novak 1978, Skemp 1971, Geldenhuys, Helm & Van Rooyen 1992.] The primaries of a CRKS may be seen as the primitive concept-names with respect to that CRKS.

From a number of specific examples (instances) of a relationship, such as

John is the father and Mary the mother of daughter Louise

each represented, as a triple in this case, by a vertex on the right, we can abstract the relationship:

The father and the mother of this daughter.

This is represented, as a triple in K in this case, by the vertex on the left. Our isomorphism finding algorithm will produce the arrows. Each arrow represents a trivial isomorphism that maps a single tuple such as <John, Mary, Louise> to the tuple <father, mother, daughter> in our illustration. In the case in which the vertices on the right represent CRKS's, "discovered" by observation, that have pairwise isomorphic sub-CRKS's, the arrows to the left represent isomorphisms that abstract a "theory CRKS" I in K as described for problem solving. The potential arrows to the right produce new examples of I. Thus examples of I are, at least partially, structurally analogous (isomorphic) with I. Theorem proofs can be displayed as CRKS's, and can be treated in a way similar to examples, abstracting the deductive basis of the proofs isomorphically. Alternatively, each theorem proof can be regarded as a CRKS associated with a vertex, in K, that is labeled with the name of that theorem. Finally, in the case in which the vertices on the right represent problem solution CRKS's the vertex on the left represents the abstracted solution technique CRKS in K and the arrows of the primitive binary relationship represent structural analogies as mentioned previously. In all five cases, once the vertex on the left has been established the learner can actively use the abstracted information: For example, once a primary concept-name such as "red" has been established the learner is able to actively seek red things. A more extensive discussion can be found in Geldenhuys, Helm & Van Rooyen [1992].

STAGES OF LEARNING IN THE CRKS VIEW

1. The *pre-linguistic* stage. Here we have the notion of an *attention* vertex. It arises by virtue of a *primitive binary relationship*, i.e. one that has a singleton co-domain, that expresses an invariant *perception* which is found by *induction* against a background of *perceptual noise*. We suggest that this is how *awareness of the environment* begins.

2.The *primitive linguistic* stage. Here we have the notion of a *primitive concept-name*. It arises by virtue of a *primitive binary relationship*, that expresses an invariant *symbol* which is found by *induction* against a background of *symbolic noise*. We suggest that this is how *language use* begins.

3.The *linguistic* stage. Here we have two sub-stages.

(i)Extension of the learner's CRKS's by discovering relationships. Here we have the notion of a *relationship:* It arises by virtue of a *primitive binary relationship* that expresses an invariant abstract *relationship* which is found by *induction* from a set of examples (instances) against a background of *relational noise*. The members of this primitive binary relationship arise from *abstraction isomorphisms.* We suggest that this is how *significant abstraction* first arises.

(ii)*Extension* of the learner's CRKS's by discovering examples, proving theorems and solving problems. Here we have the notion of an *abstract knowledge or problem solution concept-relationship knowledge structure*. It arises by virtue of a *primitive binary relationship* that expresses an invariant *concept-relationship knowledge structure* which is based on a set of examples or of theorem proofs or of instances of problem solutions, and which is found by *induction* against a background of *noise that consists of other examples or theorem proofs or problems*. The members of this primitive binary relationship are *abstraction isomorphisms*. By expressing a common problem solving technique as a CRKS in the "high capacity" memory for instance, this situation provides the learner with the potential to solve other problems of the same kind by means of algorithmic isomorphisms. We suggest that this is how the ability to *formulate and solve problems* begins. Background for these comments can be found in Geldenhuys, Helm & Van Rooyen [1992].

SOME GENERAL COMMENTS

The CRKS model can be used at all levels of education. Very briefly, this can be described as follows. At the highest level we start with a curriculum CRKS in which the vertices represent courses, and the arrows represent prerequisite and parallel conditions among those courses for example. At the next level each course corresponds with a syllabus CRKS. The interaction of this level with the uppermost one will assist the curriculum designers in making decisions about the interrelationships among the courses in the curriculum CRKS. Each concept-name in a syllabus CRKS can in turn be related to a CRKS in a prior or a parallel level, and the same applies to each non-concept-name word used in any statement in any CRKS since such a word must appear as a concept-name in some CRKS. The lowest level but one consists of the primitive CRKS's for the primitive concepts, each such CRKS establishing one primitive concept by means of examples. The lowest level is the pre-linguistic one, and consists of those primitive CRKS's each of which involves a single attention vertex. Again more detail will be found in Geldenhuys, Helm & Van Rooyen [1992].

It is evident that teaching, in the CRKS approach, relies heavily on the establishment and the precise and flexible use of a natural language because, semantically, our model is a linguistic one. To find suitable commonly understood primary concept-names from which to begin teaching [Novak & Gowin 1984, Novak 1981, Beeson 1981, Summers 1982, West & Kellet 1981, Stewart 1980] the teacher can ask the learners to draw up a CRKS showing all they initially know about the suggested primary concept-names. [See also Novak & Gowin 1984, Novak 1990, Pankratius 1990, Heinze-Fry & Novak 1990.] Our algorithm for finding structural analogies, which allows for relative permutation when seeking corresponding tuples, can be adapted to help the teacher to find a common CRKS that contains the prospective primary concept-names from the individual efforts of the learners in a class. Another standard teaching procedure is to state the objectives of a syllabus in the form of a brief overview. This can be achieved by initially presenting a sparce CRKS that displays the primary concept-names, the goals, and selected paths from primaries to goals through some key concept-names. Computer support can be used to assist curriculum designers, syllabus designers and teachers investigating presentation techniques, but in the long run human decision making remains paramount. For any reasonably small CRKS's all the techniques can be carried out manually.

In summary:

°The work of the SERP

- introduces a new knowledge structure called a Concept-Relationship Knowledge Structure [CRKS]

- concentrates on the applications of CRKS theory in the design, "weak point" analysis and hierarchical substructures presentation of syllabuses in CRKS form

- describes the formal definition, and the many uses, of analogical reasoning in teaching and learning, in terms of CRKS isomorphisms.

°The semantics of a CRKS takes the form of a list of statements of relationships, of arities ≥ 2, among concept-names in a natural language: Thus the model is linguistic. The syntax, with which one works formally (i.e. mathematically), enforces a condition of "derivability" which is equivalent to "learnability", in the Ausubel view, in the sense that all "new" knowledge is formally derived from previous knowledge. Declarative knowledge appears as statements of relationships while procedural knowledge and analogical reasoning are closely related and are dealt with in terms of CRKS isomorphisms. The theory is supported by readable diagrams and simple constructive algorithms.

°CRKS theory is essential for planning courseware and its hierarchical presentations in a formal (i.e. mathematical) science of education. It can be seen as a new facet of the use of computers in education that preceeds, but is independent of, the current CBE/CAL/CAI uses.

°The CRKS view of learning hinges on:

- the ability to abstract invariants
- the ability to learn and use a natural language
- the ability to use analogical reasoning.

The teacher's task is to

- expose the learner to appropriate learning experiences
- assist the learner to formulate the relationships discovered under the teacher's guidance.

°In the CRKS view, based on the formal definition of derivability and the hypothesis that derivability expresses learnability, it follows that

- intuitive thinking is modelled by fast access cascade searches in CRKS's
- deductive thinking is modelled by limited access cascade searches in CRKS's
- analogical reasoning, procedural learning and induction are intimately linked via the formal notion of CRKS isomorphism
- the CRKS model displays context dependence
- the existence of a concept-name at a higher, more abstract level, linked to a CRKS at lower level, implies partially hidden background knowledge that can readily be accessed. The same applies to parallel levels.
- the semantics, that is flexible and dynamic because it is expressed in a natural language, is made precise by context specification.

°Some attractive features of the CRKS approach are

- independence of computers for small CRKS's

- automatic integrity checking at each stage of design of a CRKS

- "weak point" analysis of a CRKS by means of gauges

- computer support for large CRKS's, for example computer supported improvement of the organization and integratedness of study material, and computer supported development of presentation (as opposed to interaction/communication) strategies

- CRKS's can be constructed, and therefor stored, in manageable units that are not necessarily independent

- the approach is interactive, allowing the teacher great freedom of choice within the basic design and presentation rules

- courseware in CRKS form, and presentation strategies for it, can be designed by a team of subject experts, using computer support, acting together or as individuals

- the potential for individualization, for both learners and teachers, and for student participation in the discovery and formulation of relationships

- finding "common ground", and testing by means of student designed CRKS's, hinge on the use of CRKS isomorphisms. A basic isomorphism finding algorithm has been developed.

CONCLUSION

In our understanding of the term "science", a science of education must be based on a formal model. The formal (syntactical) aspects of the CRKS approach are mathematically rich and the interpretation of the formal theory conforms well with the intuitive notions built up by educators during years of teaching experience. In addition the model has added some new and useful insights to the theory of teaching and learning [see Geldenhuys, Helm & Van Rooyen 1992]. What we see in our CRKS theory and its uses in education is the beginning of a science of education in the sense that our approach involves a model which generates predictions that can be empirically tested, resulting in useful feedback. At one level there is the prediction that the learner's CRKS is identical with the teachers prescribed syllabus-CRKS at various stages, which can be tested by asking the learner to draw up his/her current CRKS for comparison with the syllabus using the algorithm for structural analogy. At another level lie the predictions made from a known CRKS in modelling a new CRKS. While our work refers to a beginning science of education in general, it is most easily applied to science education.

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