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AN EVALUATION OF INSTRUCTIONAL INTERVENTIONS TO ERADICATE THE MISCONCEPTION OF REPRESENTATIVENESS

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ABSTRACT

Students' misconceptions are particularly problematic in learning about probability as they result in a lack of conceptual understanding of the nature of probability and probabilistic reasoning. The purpose of this research was to investigate the effectiveness of several instructional interventions intended to eliminate students' misconceptions of representativeness in learning about probability. Two different forms of a test instrument study after which a controlled experiment was conducted to evaluate the effectiveness of several instructional interventions designed to create varying levels of cognitive conflict and conflict resolution. One form of the test was used to determine students' eligibility for the experiment. Students identified as having misconceptions based on their responses to the test were assigned to one of four instructional interventions. The second form of the test was used to evaluate the effectiveness of the instructional interventions. Results of the experiment found the instructional interventions designed to create cognitive conflict and conflict resolution to be effective in long-term elimination of students' misconceptions of representativeness.

INTRODUCTION

Student knowledge often includes misconceptions and instruction can be very ineffective if such misconceptions are not addressed. Failure to recognize that students have misconceptions and tailor instruction to accordingly eliminate students misconceptions results in little progress toward improving conceptual understanding. Misconceptions must be identified prior to instruction so that they can be addressed during instruction because simply providing students with correct information does not necessarily eliminate old ways of thinking.

Students often form misconceptions of a concept through brief, informal experiences outside the classroom, or prior to entering school (Garfield & Ahlgren, 1988; Konold, Pollatsek, Well, Hendrickson, & Lipson, 1990). Teachers must be aware of students' misconceptions so that instruction is directed toward their eradication. Students are often able to combine new information they learn with their existing beliefs and misconceptions or alter new information so that it is consistent with their current understanding, making it difficult to eradicate their misconceptions during instruction.

New information that may conflict with students' existing knowledge. Teachers can help students resolve this conflict by clearly delineating new concepts, contrasting them with common misconceptions. Students must not only understand the new concept, but also understand why any prior misconceptions they held are incorrect. If the student fails to resolve the conflict, the students' misconceptions will persist.

Instruction that specifically addresses students' misconceptions should be more effective than instruction that does not take possible misconceptions into account. The purpose of this study was to evaluate the effectiveness of several instructional interventions designed to eliminated misconceptions of representativeness in learning probability. Misconceptions of probability are particularly difficult to eliminate during instruction as they appear to be of a psychological nature and are strongly held (Konold, 1989a, 1991; Shaughnessy, 1977, 1981). This is probably due to the nature and development of probabilistic reasoning. There are strong, experiential differences between the development of misconceptions in probability and misconceptions in other sciences.

Many students do not understand the laws of probability but have developed their own way of reasoning about uncertain events (Kahneman & Tversky, 1972, 1973; Konold, 1989b, 1991; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1971). Students' lack of understanding may be due to a lack of experience with the mathematical laws of probability or because they use heuristics (Kahneman, Slovic, & Tversky, 1982; Kahneman & Tversky, 1972; Tversky & Kahneman, 1974). Heuristics are strategies based on intuition or personal beliefs that are used to reduce the complexity of a problem.

People use heuristics to estimate complicated probabilities which often result in accurate or very reasonable estimates. Successful use of heuristics is very reinforcing and as a consequence they are often applied automatically. The use of heuristics can also be misleading, causing misconceptions in how people think about probability. Research has shown that even experts in probability unconsciously use heuristics in some situations (Tversky & Kahneman, 1971).

Types of misconceptions in probability have been investigated and characterized by a number of researchers (Kahneman & Tversky, 1972, 1973; Konold, 1989a, 1991; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1974). Several different types of misconceptions have been identified and classified according to the conceptual misunderstandings they represent (Garfield & Ahlgren, 1988). Among these misconceptions are those of "representativeness", "availability", and inferring causation from correlation. The misconception of representativeness was selected for investigation because it has been consistently identified and characterized by previous research (Konold, 1989a; Konold, 1990; Kahneman & Tversky, 1972, 1973; Shaughnessy, 1977, 1981). Representativeness is a heuristic used to estimate the probability of uncertain events by relying on the degree to which a sample or event reflects the population of such events. It includes a misinterpretation of the law of large numbers and an insensitivity to prior events or sample size. For example, most people think that when tossing a coin, a sequence of six tails (TTTTTT) is less probable than a mixed sequence such as THHTHT because *all tails* is not representative of the distribution of events. A randomly-mixed sequence of heads and tails (e.g., THHTHT), is considered representative because this sequence looks more like the theoretical distribution which comprises 50% tails and 50% heads. Another example of representativeness is that most people believe the seventh toss of a coin is more likely to be a head than a tail when the first six tosses were all tails. Such a belief results when people interpret the law of large numbers to say that eventually distributions should balance out and there will be an even mixture of heads and tails. Much is already known about the nature and pervasiveness of representativeness and researchers have already begun developing test materials to identify the existence of this misconception (Konold, 1989a, Hirsch, 1993) but little work been done to establish instructional interventions to eradicate the misconception of representativeness during classroom instruction.

Instructional Interventions to Eliminate Misconceptions

Instructional interventions designed specifically to address misconceptions are necessary because research has shown that misconceptions are not often eliminated during regular classroom instruction. There are surprisingly few instructional interventions designed to eliminate students' misconceptions in the area of probability, although numerous instructional interventions have been developed for use in other disciplines (Garfield & Ahlgren, 1988; Shaughnessy, 1992). Instructional interventions that have successfully eradicated misconceptions have all been characterized by what Novak (1977) calls "cognitive dissonance", the experience of contradictory concepts or events. Whenever a student experiences a concept meaning that appears to be in direct conflict with his or her current understanding of the concept, there can be a negative emotional response or experience of dissonance that causes students to become dissatisfied with their current understanding. Novak suggests that this cognitive dissonance be resolved by comparing and contrasting the discrepant concepts until students are able to reject the incorrect concept meaning in favor of the correct meaning. Instructional interventions of this type, developed to create cognitive conflict and promote conflict resolution for the purpose of eradicating students' misconceptions, have been more successful in science, physics and mathematics than similar interventions developed in probability and statistics.

Activity-based group instruction, computer simulation, redesigning textbooks, and activating students' prior knowledge have all been shown to be effective methods of creating cognitive conflict and the resolution of this conflict. A successful intervention outlined by Shaughnessy (1977) made students aware of 1) the misuses of statistics and the confounding role of their personal beliefs and 2) the conflict between their own understanding of a concept and the correct understanding of the concept, through small group activities that simulate the phenomenon being studied. Shaughnessy asked students to make predictions before carrying out group activities so that by the end they had to resolve the conflict between their own beliefs and the results of the experiment. The intervention was successful in eliminating students misconceptions but it is not possible to conclude which aspect(s) of the intervention are necessary.

Creating Cognitive Conflict

For subjects other than probability, the creation and resolution of cognitive conflict appears to be a necessary and sufficient characteristic for instructional interventions to be successful in eradicating students' misconceptions. Misconceptions in probability and probabilistic reasoning appear to be more difficult to eliminate because of their psychological and intuitive nature (Falk & Konold, 1990). Students may need to experience the conflict themselves rather than hear an explanation from the teacher. Interaction among students and peer confrontation may help create and resolve the conflicts necessary to eliminate students' misconceptions of probability.

When students encounter inconsistencies between their present understanding and the correct understanding they have an opportunity to learn (Furth, 1981). It is not known whether this learning must take place through trial and error so that students experience for themselves how their understanding differs from correct understanding (Fischbein & Gazit, 1983; Shaughnessy, 1977, 1981), or if it would be sufficient for teachers to demonstrate the misunderstanding for students by contrasting common misconceptions with correct information in such a way as to create conflict in the mind of the students and then help the students resolve the conflict.

This study examined three possible methods of creating and resolving cognitive conflict were compared: direct instruction, individual activities, and small group activities in which students had varying degrees of misconceptions about probability. In direct instruction, the correct concepts related to the lesson are explained and students are given an explicit explanation of why their misconceptions are wrong. Students may simply accept the new information provided by the teacher, rather that explore the subject matter more closely to understand why their understanding of the concepts is not correct. Learning under these conditions does not always require the cognitive restructuring required to understand information that conflicts with prior knowledge. A learning environment that allows students to interact more closely with the material may be more effective. Students may need to resolve for themselves the inconsistencies between what they think and what they actually experience. Using classroom activities can be an effective method of encouraging students to become more actively involved in the lesson they are trying to learn.

Cognitive approaches to learning also support activity-based learning (Anzai & Simon, 1979). When students are required to participate in an activity as part of the learning experience they may become more active processors of the information. This can be beneficial because the information is processed at a deeper level. The depth of processing theory of information processing suggests that when information is processed at deeper levels it is better remembered and understood (Anderson, 1985). When students become actively involved in a learning experience information may be elaborated and rehearsed. As a consequence, students may be more likely to notice and investigate inconsistencies between their prior knowledge and the concept currently being explored. In addition, students engaged in a problem-solving activity are able to investigate results they do not understand with the option of repeating the steps in the activity that led to the results they did not understand. Repeating an activity is a form of elaboration and rehearsal. Information that is elaborated and rehearsed during the learning process is better remembered and understood. Under these conditions students are more likely to develop problem-solving strategies that can be used to solve new problems because they learned and developed a procedure rather than memorize large amounts of information (Bransford, Franks, Vye, & Sherwood, 1989).

The interactions and group discussions that result when students are required to work together toward a common solution to a problem have been found to be more effective than individual problem solving (Sharan, 1980; Slavin, 1980). This is especially true when the task requires the resolution of controversy (Johnson, Brooker, Stutzman, Hultman, & Johnson, 1985; Johnson & Johnson, 1985; Smith, Johnson, & Johnson, 1984). Research has shown that heterogeneous groups of students with varying levels of prior knowledge or ability gain more during the learning process than students who worked individually or in homogeneous groups (Nijhof & Kommers, 1985; Tudge & Rogoff, 1989; Webb, 1985). Vygotsky's notion of the zone of proximal development suggests that less capable students working with more capable students promotes an atmosphere of motivation that results in improved understanding of the problem (Vygotsky, 1978). The more capable students must understand the needs of the less capable student and the less capable student strives to attain a more advanced understanding of the problem to be solved. This suggests that small-group activities intended to teach students about the laws of probability and eliminate misconceptions will be most effective if at least one group member does not hold misconceptions of probability.

Research Questions

The following questions concerning students' misconceptions of representativeness were addressed in this study:

1. Is the relationship between the presence or absence of misconceptions related to representativeness following instruction influenced by the instructional intervention used to teach probability?

2. Is the presence or absence of misconceptions related to representativeness following instruction related to the creation and resolution of cognitive conflict during instruction?

3. Is the presence or absence of misconceptions related to representativeness following instruction influenced by the methods used to create the cognitive conflict during instruction?

METHOD

Participants

Participants were 109 undergraduate students recruited from psychology and educational psychology classes at a number of colleges in New Jersey during the Spring, Summer, and Fall semesters of 1992. Participants received course credit in return for their participation.

Materials

Tests of Representativeness. Students took an 10-item test developed by Hirsch (1993), to identify misconceptions of representativeness, before and after participating in one of four instructional interventions. Two different forms of the test instrument, called the pre-test and post-test, are available. The difficulty level of the items on each test are equivalent and both tests were equally reliable. A Kuder-Richardson measure of internal consistency for dichotomously scored items found the reliability of the pre-test and post-test to be .82 and .81, respectively.

Two items on each test were open-ended questions and were included to determine if students had the basic knowledge of probability required to answer the more difficult multiple-choice questions. The remaining eight items were presented in a multiple-choice format, each with two parts.

The open-ended questions required students to calculate the probability of a simple event from a distribution of equally likely events. The first part of each multiple-choice item asked students for an assessment of probability. Student were presented with several possible outcomes and were asked "Which event is most likely?" or "Which event is least likely?":

> If a fair coin is tossed six times which of the following ordered sequence of heads (H) and tails (T), if any, is LEAST LIKELY to occur?
>
> a) H T H T H T
> b) T T H H T H
> c) H H H H T T
> d) H T H T H H
> e) all sequences are equally likely

A student who identifies c) H H H H T T as being "least likely" is thought to hold a misconception of representativeness that the result of repeatedly tossing a coin must be a random mixture of heads and tails. If students calculated the probability of each event correctly, they would have found that Option e ("all sequences are equally likely"), is correct.

The second part of each item asked students to identify a specific reason for their answer to the first item in the pair. Common explanations for misconceptions of representativeness, identified in previous research through clinical interviews with students, were used to construct the second part of each item. The item in the previous example was followed on the test by:

> Which one of the following best describes the reason for your answer to the preceding question.
>
> a) There ought to be roughly the same number of tails as heads.
> b) Since tossing a coin is random you should not get a long string of heads or tails.
> c) Every sequence of six tosses has exactly the same probability of occurring.
> d) Since tossing a coin is random the coin should not alternate between heads and tails.
> e) Other

The distractors used were designed specifically to confirm that

1) students who gave correct answers to the first item gave the answer for the right reason and 2) students who gave answers that indicated misconceptions of representativeness also provided a reason for the answer that indicated a misconception of representativeness. For example, a student who identified Option b (H H H H T T), in the first part of the question, as being "least likely" should also identify

Option b (Since tossing a coin is random you should not get a long string of heads or tails) as the reason for their answer if they hold misconceptions of representativeness. The correct answer to the first part of the question is "All sequences are equally likely" (e). In order to demonstrate understanding of the underlying probability concept, respondents should also select reason c) "Every sequence of six tosses has exactly the same probability of occurring" as the answer to the second part of the question.

Teaching Materials

Lecture. Participants received a 28-minute, video-taped lecture as an introduction to the laws of probability. The lecture covered definitions of terms necessary to understanding probability, counting techniques, sample space, the classical and frequency definitions of probability, how probabilities are calculated and interpreted, the concept of equally likely events, independence and the effects of sample size on the probability of sequences of events.

Activities. Two of the four instructional interventions included activities designed to confront students with their misconceptions of representativeness. The first activity required students to draw colored marbles at random from a small bag, repeatedly. The second activity simulated a game of chance similar to a Lottery. Participants were asked to pretend to buy lottery tickets after which they held a mock drawing to find a winner.

Procedure

The study was conducted in three sessions. Session I was a screening session to determine whether participants where eligible for the remaining two sessions. The remaining two sessions were held in small groups at various times throughout the semester. Instructional interventions were randomly assigned to time intervals within the constraint of balancing the number of students who participated in each instructional intervention.

Session I. Participants took the ten item pre-test. Eligible participants were all participants who showed evidence of misconceptions on the pre-test and a small random sample of those who did not. Participants who did not hold misconceptions were needed for the small-group, activity-based instructional intervention in Treatment IV which will be explained later.

Session II. The second session lasted an hour and ten minutes. Each of the four instructional interventions, Treatments I, II, III and IV, were intended to teach students the same material. The methods used to process the material varied for each of the treatments. Participants in all four treatments viewed the video-taped lecture. After the lecture there was a 25 minute post-lecture session.

Treatment I served as a control. The lecture was followed by a question and answer period during which the teacher posed the following hypothetical situation(s):

"Suppose you have a bag that contains 6 marbles: 2 yellow,
2 red and 2 green. You reach into the bag without

looking and pick one of the marbles at random. What is the chance that the marble you pick is red?"

"Let's say when you reached into the bag the marble you pick is yellow. You put it back in the bag, mixed them up, and pick again. What is the chance that you would pick a red marble the second time?"

"What about another yellow one? What is the chance of picking a yellow marble the second time if you already picked a yellow one the first time?"

After participants had time to think and record their answers on their answer sheets, the teacher reviewed the questions and explained the correct answers. Some of the questions required the participants to make predictions about the outcome of the problem. No attempt was made by the teacher to point out misconceptions of representativeness or help the students resolve any conflict that may have arisen between the students' prior knowledge about probability and the correct concepts presented in the lecture.

Treatments II, III, and IV were designed to confront students with their misconceptions related to representativeness, and to help students resolve the conflict between their misconceptions and the correct concepts taught in the lesson. Participants in Treatments II were asked the same questions as in Treatment I and asked to make predictions about the outcome of the same problems. Misconceptions of representativeness were also discussed. The teacher contrasted misconceptions of representativeness with the ideas of independence and equally likely events while reviewing the questions and explaining the correct answers.

The discussion of misconceptions included examples of the typical kinds of answers provided when people have misconceptions of representativeness. These answers were contrasted with the correct concepts. The following quotation was part of the explanation provided by the teacher:

> "Very often people think that the chance of picking a different color marble on the second pick is greater or that the chances of picking the same color marble two picks in a row is smaller because they think that the colors should some how 'balance out'. It makes sense to think this way because if you actually continued to pick marbles out of the bag, look at the color, put the marble back in the bag, mix them up and pick again, after a large number of picks you would probably notice that the distribution of colors does balance out <u>BUT</u> that does not mean that the marbles know which color should come next so that the colors will balance out. Each individual pick is independent of all previous picks. Even if you pick a red marble three times in a row, if you put the marble back in the bag each time, mix them up, and pick again there is no reason to believe that the chance of picking a red marble again is any different than it was before. If there are three different color marbles in the bag, and there are the same number of marbles of each color in the bag, each time you pick the chance of picking a marble of any particular color is 1/3."

Treatment III was the same as Treatment II with the exception that, instead of seeing problems worked out on a flip chart in the front of the room, participants conducted individual activities to simulate the same problems. The teacher questioned participants asking them to make a prediction about the results and discussed misconceptions of representativeness as in Treatment II.

Treatment IV was the same as Treatment III except that instead of conducting individual activities to simulate the problem participants worked in groups of four to conduct the same activity. Groups were formed so that each group had three members with misconceptions of representativeness and one member without misconceptions. Participants in Treatment IV who held misconceptions of representativeness were assigned to groups of three by the teacher prior to the beginning of session II and then a participant without misconceptions was randomly assigned to each group. This was done in order to help create conflict within each of the groups during the post-lecture instructional activities used in Treatment IV. Participants in Treatment IV were asked to make the same predictions about the results of the activity they were asked to conduct but participants in Treatment IV were instructed to discuss their answers with the other members of their group and work together to perform the activities. The teacher reviewed the correct answers to the questions with group members after they completed the activities and discussed misconceptions of representativeness as in Treatments II and III.

<u>Session III</u>. During Session III, held exactly one week after session II, there was a 20 minute lesson presented by the teacher. This lesson followed the same format as the lesson given during Session II. The examples and/or activities utilized in Session III were slightly more complex

than those presented in the video-taped lecture and Session II. All four treatment groups received the same lesson, but the presentation of the lesson within each treatment followed the same format as in Session II.

At the end of session III, participants in all treatments were given a post-test to determine whether their misconceptions were eliminated during instruction. Test results obtained from the participants in Treatment IV that did not hold misconceptions of representativeness prior to instruction were not included in the final data analysis. These students were of no interest in this particular study because they did not hold misconceptions of representativeness prior to instruction and results of their post-test scores indicate they still did not hold misconceptions of representativeness after participating in Treatment IV.

RESULTS

The dependent measure of interest was a count of the number of participants in each treatment classified as having misconceptions after completing instruction. For this reason, hypotheses were tested using chi-square tests of independence.

Tests of Significance

A four by two chi-square test of independence was used to test the null hypothesis that there is no relationship between the instructional interventions and the presence or absence of misconceptions related to representativeness.

Results of the chi-square test do not provide sufficient evidence to reject the null hypothesis, X^2 (3, N = 103) = 3.94, $p>.05$ (see Table 1).

A two by two chi-square test of independence was used to test the null hypothesis that the presence or absence of misconceptions is not related to the creation and resolution of cognitive conflict. This test compared Treatment I with Treatments II, III and IV. This comparisons evaluates the effectiveness of instructional efforts to create cognitive conflict and conflict resolution in eradicating students misconceptions related to representativeness. Results of the chi-square test do not provide sufficient evidence to reject the null hypothesis, X^2 (1, N = 103) = 1.70, $p>.05$ (see Table 2).

Table 1

Instructional Intervention and the Presence or Absence of Misconceptions Following Instruction.

Instructional Intervention	Student has Misconceptions: No	Yes		
Treatment I	23	4	27	26.2%
Treatment II	25	2	27	26.2%
Treatment III	22	3	25	24.3%
Treatment IV	24		24	23.3%
	94	9	103	100.0%
	91.3%	8.7%		

X^2 (3, N = 103) = 3.94, p>.05

Table 2

Instructional Efforts to Create Cognitive Conflict and Conflict Resolution and the Presence or Absence of Misconceptions Following Instruction

Student had Misconceptions Following Instruction	Treatment Included Cognitive Conflict: No	Yes		
No	23	71	94	91.3%
Yes	4	5	9	8.7%
	27	76	103	
	26%	74%		100.0%

X^2 (1, N = 103) = 1.70, p>.05

A two by two chi-square test of independence was used to test the null hypothesis that the presence or absence of misconceptions is not related to the use of activities in instructional efforts to create cognitive conflict and conflict resolution. This test compared Treatment II with Treatments III and IV. This comparison examined the effect that performing activities has on cognitive conflict and conflict resolution in eradicating students misconceptions. Results of the chi-square do not provide sufficient evidence to reject the null hypothesis, X^2 (1, N = 76) = 0.05, p>.05 (see Table 3).

Table 3

The Use of Instructional Activities to Create Cognitive Conflict and Conflict Resolution and the Presence or Absence of Misconceptions Following Instruction.

Student had Misconceptions Following Instruction	Treatment Included Cognitive Conflict and an Activity			
	No	Yes		
No	25	46	71	93.4%
Yes	2	3	5	6.6%
	27 51%	49 49%	76	100.0%

X^2 (1, N = 76) = 0.05, p>.05

A two by two chi-square test of independence was be used to test the null hypothesis that there is no relationship between the presence or absence of misconceptions related to representativeness and the type of activity (individual or group), used in instructional efforts to create cognitive conflict and conflict resolution. This test compared Treatment III with Treatment IV to determine any possible differences in the effects of using individual verses group activity in instructional efforts to create cognitive conflict and conflict resolution to eradicate students misconceptions related to representativeness. While results of the chi-square test approach conventional levels of statistical significance (p=.08) there is insufficient evidence to reject the null hypothesis, X^2 (1, N = 49) = 3.07 (see Table 4).

Table 4

Individual vs. Group Activities in Creating Cognitive Conflict\

Conflict Resolution and the Presence or Absence of Misconceptions

Following Instruction

Student had Misconceptions Following Instruction	Type of Activity: Individual	Group		
No	22	24	46	93.9%
Yes	3		3	6.1%
	25 51%	24 49%	49	100.0%

X^2 (1, N = 49) = 3.07, .01>p>.05

Follow-up Testing

A small subset of participants during the 1992 Fall semester ($\underline{n}$=27) agreed to take a follow-up post-test several weeks after completing instruction. Another six students who were pre-tested and identified as having misconceptions of representativeness but did not participate in a instructional intervention also took the follow-up post-test for a total of 33. The follow-up post-test was identical to the first post-test.

The six students who did not participate in a treatment intervention all provided answers that indicated misconceptions of representativeness on the follow-up post-test. Of the twenty-seven students who did participate in one of the four treatment interventions only three provided answers that indicated misconceptions of representativeness on the follow-up post-test. A chi-square test of independence was performed to test the null hypothesis that there is no relationship between participation in an instructional intervention and the presence or absence of misconceptions related to representativeness. Results of the chi-square test provide sufficient evidence to reject the null hypothesis, X^2 (1, $\underline{N}$ = 33) = 19.56, $\underline{p}$<.001. The presence or absence of misconceptions related to representativeness appears to be related to students' participation in one of the four instructional interventions (see Table 5).

The three students who participated in a treatment intervention and provided answers on the follow-up post-test that indicated they still held misconceptions of representativeness were in Treatment I where no attempt was made to create cognitive conflict or conflict resolution. None of the participants in Treatments II, III or IV provided answers that indicated misconceptions of representativeness on the follow-up test (see Table 6).

Table 5

Participation in an Instructional Intervention and the Presence or Absence of Misconceptions

Student had Misconceptions Following Instruction	Student Participated in a Treatment Intervention: No	Yes		
No	0	24	24	72.7%
Yes	6	3	9	27.3%
	6	27	33	
	18%	82%		100.0%

X^2 (1, N = 33) = 19.56, $p<.001$

Table 6

Distribution of Participants Who Took the Pre-Test and the Follow-up Post-Test

	# who took the Follow-up Post-Test	# identified as having misconceptions
No Treatment	6	6
Treatment I	7	3
Treatment II	7	0
Treatment III	6	0
Treatment IV	7	0

Of the three participants in Treatment I that took the follow-up post-test and provided answers that indicated misconceptions of representativeness, one gave answers that indicated misconceptions of representativeness on the first post-test also. Based on this information, it is reasonable to conclude that, of the 27 students who participated in an instructional intervention and took the follow-up post-test, 26 of them learned the correct concepts of probability, at least temporarily. They provided correct answers to all seven pairs of multiple-choice items intended to identify misconceptions of representativeness, and the two open-ended items, on the first post-test.

The student who provided answers to the multiple-choice items on the first post-test that indicated misconceptions of representativeness did not answer one of the open-ended questions correctly. This student did not appear to have learned the correct concepts of probability and was, therefore, dropped from further analysis of the follow-up data.

A two by two chi-square test of independence was used to test the null hypothesis that the presence or absence of misconceptions of representativeness following instruction is not related to instructional efforts to create cognitive conflict and conflict resolution. This comparison is different than the original comparison made between Treatment I and Treatments II, III and IV because it examines the effectiveness of cognitive conflict and conflict resolution in eliminating students' misconceptions several weeks after completing instruction rather than immediately following instruction. Results of the chi-square test provide sufficient evidence to reject the null hypothesis, X^2 (1, N = 26) = 7.22, p<.01. Contrary to previous conclusions, it appears that Treatments II, III and IV, designed to create cognitive conflict and conflict resolution, are more effective in eliminating students' misconceptions of representativeness than Treatment I which was not intended to create cognitive conflict and conflict resolution. The difference appears to be in the timing of the evaluation (see Table 7).

Table 7

Instructional Efforts to Create Cognitive Conflict and Conflict Resolution and the Presence or Absence of Misconceptions Several Weeks After Completing Instruction

Student had Misconceptions on Follow-up Post-Test	Treatment Included Cognitive Conflict: No	Treatment Included Cognitive Conflict: Yes		
No	4	20	24	92.3%
Yes	2	0	2	7.7%
	6 23%	20 77%	26	100.0%

X^2 (1, N = 26) = 7.22, p<.01 (Fisher's Exact Test)

Discussion

All of the instructional interventions were effective in eliminating students' misconceptions of representativeness, at least temporarily. When the first-post test was administered, immediately following instruction, only 9% of the participants in any of the instructional interventions appeared to still hold misconceptions of representativeness and there appeared to be no significant differences in treatment efficacy. It was therefore, not possible to conclude that the instructional efforts to create cognitive conflict and conflict resolution during instruction (Treatments II, III and IV) were more effective in eliminating students' misconceptions of representativeness than the instructional intervention that was not designed to create cognitive conflict and conflict resolution (Treatment I).

Results of the follow-up post-test, administered several weeks after instruction, indicate that there may be long-term benefits of instructional efforts to create cognitive conflict and conflict resolution for the purpose of eliminating students' misconceptions of representativeness that are not evident immediately following instruction. When a group of participants were re-tested several week later, some of the participants in Treatment I (the control group that received instruction not intended to create cognitive conflict and conflict resolution) provided answers that indicated they still held misconceptions of representativeness. The fact that these participants responded correctly to the items on the post-test immediately following instruction but were not able to respond correctly to exact same questions several weeks later is an indication that under the conditions created in Treatment I, learning may only have been temporary.

There is a need for effective methods of instruction to improve students' conceptual understanding of probability and probabilistic reasoning. The study presented here was conducted to evaluate the effectiveness of instructional efforts to create cognitive conflict and conflict resolution to eliminate students' misconceptions of representativeness. Three of the treatments were designed to create varying levels of cognitive conflict during instruction: direct instruction, individual activities and small group activities in which students had varying degrees of misconceptions about probability. The fourth instructional intervention served as a control. The control intervention covered the same subject matter as the other three interventions, using the same basic lesson plan, but no attempt was made to induce cognitive conflict or conflict resolution.

From the results of the first post-test, administered immediately following instruction, it was obvious that all of the instructional interventions were effective in eliminating the participants' misconceptions of representativeness. Ninety-seven percent of the participants in all four treatment interventions no longer appeared to hold misconceptions of representativeness and there were no observable differences in treatment efficacy. Therefore, it was not possible to conclude that the three instructional interventions designed to create cognitive conflict and conflict resolution were more effective in eliminating students' misconceptions than the instructional intervention that was not designed to create cognitive conflict and conflict resolution.

There are several possible explanations for why even the control intervention was effective in eliminating the participants' misconceptions of representativeness. First, all participants viewed the same video-taped lecture and the quality of the instruction they received was good. The lecture was presented by an experienced professor. The correct concepts of probability and the steps required to calculate the probability of simple events were clearly delineated. The instructor dealt with concepts of probability that are particularly vulnerable to misconceptions. Most instruction is not necessarily designed to consider students' misconceptions.

Second, it is possible that requiring the participants to record their answers to the questions asked by the teacher during the post-lecture activities was enough to create cognitive conflict for participants, including those in Treatment I. Eventually, the teacher provided the correct answer for each of the questions and participants had the opportunity to realize that their answers may have been different. At this point it is not possible to reject the hypothesis that instructional efforts to create cognitive conflict and conflict resolution are more effective in eliminating students misconceptions than the control treatment. The format of the post-lecture used in all four treatments may have been effective in creating cognitive conflict. As the lecture focused on information related to misconceptions, students in the control condition may have also experienced cognitive conflict. Alternatively, students in Treatments II, III, and IV may not have actually experienced cognitive conflict despite the manipulation of instruction.

Twenty-five percent of the participants, who did not appear to have misconceptions based on the results of the post-test, returned several weeks after completing instruction to take a follow-up post-test. The follow-up post-test was identical to the post-test they had taken at the end of session III, immediately following instruction. Some of the participants in Treatment I (the control group) that received instruction not intended to create cognitive conflict and conflict resolution provided answers that indicated they still held misconceptions of representativeness. The fact that these participants responded correctly to the items on the post-test immediately following instruction but were not able to respond correctly to exact same questions several weeks later suggests that under the conditions created in Treatment I, learning may only have been temporary.

The first post-test was administered to participants immediately following instruction, at the end of session III.

Possibly, participants were able to respond correctly to the items on the test because they still held the necessary information in working memory (Anderson, 1985). Perhaps the participants in Treatment I were able to acquire sufficient declarative knowledge of probability to answer the questions on the post-test without actually understanding the concepts of probability. To provide the correct responses again several weeks later, when the exact same test was administered as a follow-up evaluation, participants were required to recall the information from long-term memory. The only participants who were not able to recall the correct information were in Treatment I, the control group.

When students do not fully understand the concepts being taught during a lesson it is not possible for them to encode the information effectively in long-term memory. To be effective, information stored in long-term memory should be meaningful (Anderson, 1985). Information that is not fully understood when it is encoded in long-term memory may not be very meaningful or useful when an attempt is made to recall the information. The participants in Treatment I may have been unable to recall the information several weeks later because they did not fully understand the correct concepts of probability taught during instruction.

As participants in Treatment I were not able to respond correctly to questions on the follow-up post-test it is possible to conclude, at least tentatively, that the treatment interventions intended to create cognitive conflict and conflict resolution where more effective than the control intervention in helping participants understand the correct concepts of probability. The hypothesis that all participants experienced some degree of cognitive conflict during instruction may explain why all four treatment interventions appeared to be equally effective in eliminating misconceptions immediately following instruction and why the three instructional interventions designed to create cognitive conflict and produce conflict resolution appeared to be more effective long-term. The three instructional interventions designed to create cognitive conflict and produce conflict resolution made specific reference to common misconceptions of representativeness. The discussion of misconceptions of representativeness in which the teacher contrasted misconceptions of representativeness with the correct concepts of probability, may have been effective in helping the participants resolve the conflict between their current misunderstandings of probability and probabilistic reasoning and the correct concepts taught during the lesson. Participants in Treatment I did not have the benefit of this discussion and therefore may have had difficulty resolving the conflict on their own hindering any long-term conceptual understanding of probability and probabilistic reasoning. Students in a typical statistics classroom do not have the benefits of deliberate attempts to create conflict during instruction or a discussion of misconceptions which explains why many college students hold misconceptions about probability despite having received classroom instruction.

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