Third Misconceptions Seminar Proceedings (1993)

Paper Title: Some difficulties of students with the translation of word statements into mathematical symbol language and with manipulation of mathematical relationships
Author: Jiya, Z.

Abstract: The idea that Physics students must have a mastery of basic mathematical tools has been with us for a long time. In physics even the simplest information requires some conceptual framework to enable it to be assimilated. Simple mathematical ideas like proportion, functional relationships, ratio etc are involved in many of the physical concepts eg pressure and volume, spring force, density etc; and these exercise a powerful organising influence in helping to understand the physical concepts and in the construction of knowledge.

Keywords: Concept Formation,,,Scientific Concepts,Math Concepts,,,,
General School Subject: Physics
Specific School Subject:
Students: College Freshmen

Macintosh File Name: Jiya - Mathematics
Release Date: 9-11-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

by the Novak-Golton Fund, which is administered by the Department of Education at Cornell University. If you have found this collection to be of value in your work, consider supporting our ability to support you by purchasing a subscription to the collection or joining the Meaningful Learning Research Group.
----- -----

Some difficulties of students with the translation of word statements into mathematical symbol language and with manipulation of mathematical relationships:

Z. Jiya:Department of Physics
University of Fort Hare
Alice; 5700 South Africa

INTRODUCTION:

The idea that Physics students must have a mastery of basic mathematical tools has been with us for a long time. In physics even the simplest information requires some conceptual framework to enable it to be assimilated. Simple mathematical ideas like proportion, functional relationships, ratio etc are involved in many of the physical concepts eg pressure and volume, spring force, density etc; and these exercise a powerful organising influence in helping to understand the physical concepts and in the construction of knowledge.

Some attempts have been made to correlate mathematical skills and success in physics. These investigations have confirmed long standing suspicions that

(i) Even mathematically sophisticated college students were sometimes unable to translate accurately between English and Mathematical Equations [Lochhead: J of Mathematical Behaviour 3(10) 1980].

(ii) Inadequacy of Mathematical skills is more a predictor of failure than a predictor of success. A high score on the mathematical test is not a guarantee for success; but unless the student has the {adequate} mathematical skills, his performance in Physics will be poor [Hudson and Mc Intire: Am J of Phys 45(5) May 1977].

(iii) In some situations even seemingly adequately prepared pre-service teachers [mathematics and physics majors] perform unsatisfactorily in tasks where they are tested for conceptions on proportionality [Yap K Chin: Int J of Science Education August 1992].

At this very conference there will probably be a couple of papers which seek to highlight some of the issues raised above on the local scene. Very recently in my own work experience more than 40% of a class [N = 115] made an incorrect substitution for t in the following equation [only 35% did it right].

y = A Cosine wt   t =y/A Cosine w

Our experience has been that inappropriate Physics conceptions [intuitive ideas, idiosyncrasies] exacerbate this problem. If the student, for some reason outside the mathematical relationship, believes differently as to how a physical system will behave the problems are even larger - this will be illustrated later on in the paper.

At the beginning of the year [1992] an effort was made to quantify some of these issues in a simple way

(i) to attempt to isolate some fundamental problems with mathematical manipulation and translation in physical situations where the conceptual framework of the problem is a simple one, or definitive physical concepts are removed; and

(ii) to establish how far the manipulation of equations affects learning in some actual physics knowledge contexts.

R E S U L T S:

(a) Initially a simple test was given at least 6 [six] weeks into the term - when we perceived that the students had settled down into the routine of their studies. The time allowed was 30 minutes and only a few students actually asked for more time. The students tested were the Physics I Ancillary course at the University of Fort Hare in 1992. The number in the class was 115; and at the end 101 scripts were handed in.

Some selected Questions and Responses:

Questions selected from the Test:

1. Given that $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ f = ?

		Responses	
A.	(vu)/(u+v)	{38}	---->
B.	v + u	{41}	
C.	(u+v)/(uv)	{14}	

D. (u+v)/2 { 4}
E. vu { 2}

3. **B**
A ----------------------------------- . ------------------ C

Given that AB = L and AC = y how can we express BC in terms of L and y BC =

Responses

A. BC = y - L {81}
B. BC = y + L {18}

5. Consider the relationship explained below:
A bus is 8 metres longer than a car. Length of bus is **b** and length of car is c

Responses

A. b = c + 8 {48} ---->
B. b = 8 c {28}
C. b + 8 = c {11}
D. c = 8 b { 8}
E. y = (b +8) + c { 6}

6. Consider the relationship described below and express it in the form of a mathematical equation.
In the Faculty of Science there are **9** times as many students as there are teachers. Number of students is **S** and number of teachers is T

Responses

A. s = 9 T {40} ---->
B. 9 s = T {42}
C. 9 s > T { 7}
D. y = 9 s + T { 5}
E. 9 s + 9T { 2}
F. s > 9T { 2}
G. 9 s + T = 0 { 2}

DISCUSSION:

The examples cited above illustrate in a simple way what we have observed in most of the other sections of the general physics course. The responses in connection to the "**Teachers and Students**" were revealing and compare very much with work done by other researchers [Clement 1981] especially in relation to the error called the **reversal error** [9s = T]. The percentage of students who got this one wrong is too high. Close examination / analysis reveals that students apply mathematical techniques to solving problems rather mechanically, without thinking about the nature and {math} logic of resolving the mathematical problem. To convince them successfully that a particular mathematical rigour / procedure is inappropriate [eg the **u, v** and **f** cannot just be flipped around] is a different matter altogether.

Here is a rather pertinent example which we have come across more recently. The problem as set out indicates how many -too many- of our students resolve a difficulty met quite often in a physics problems on projectile motion:

$x = (v_o \text{ Cosine}Q)\ t$ $\qquad y = (v_o \text{ Sine}Q)\ t - 1/2\ g\ t^2$

$x = 40$ metres $\qquad y = -15$ metres $\qquad Q = 53^o$

$40 = v_o\ t \text{ Cosine } 53^o$ $\qquad -15 = v_o\ t \text{ Sine}53^o - 1/2\ g\ t^2$

$66.45 = v_o\ t$ --- (i) $\qquad -15 = vo\ t\ (0.8) - 4.9\ t^2$

$-15 = v_o\ t\ (0.8) - 4.9\ t^2$

$13.89 = t^2$

Substituting in equation (i) above

$v_o\ (3.7) = 66.45$

v_o 17,96 metres per sec

Our experience has been that is is very difficult to convince students that it is incorrect to substitute for $v_ot = 66.45$ whilst leaving the t^2 to solve for it later. To escape from this quickly - whch is what teachers often want to do - we normally rely on just pointing out that "**it is incorrect to do do it like this**" As teachers we are unable `quickly and easily' to convince the students that the procedure is not mathematically logical or systematic.

(b) In the part below an analysis of a selection of questions in a questionnaire with a physics content base is presented. The questions were asked on two consecutive years [1989 and 1990]. Close scrutiny of the responses of students confirms that students are not able to manipulate equations correctly even in situations where rather strong verbal and written cues are indicated; for example, the relevant equations were actually given in most of the situations. Inappropriate conceptions lead the students away from correct solutions. The analysis also shows that the students do not focus fundamentally on the mathematics they use, perhaps again because we as teachers do not focus on it ourselves.

Selection of some of the Questions:

1. The force of attraction between any two point charges can be doubled by:

Responses [N=325 and N=211]

		Percentage	
		1989	1990
A.	Halving the distance between them	56.6	56.9
B.	Doubling the distance between them	6.2	8.1
C.	Doubling the charge on both of objects	18.2	16.9
*D.	Doubling the charge on one of the object	15.4	14.6
E.	[students who did not choose any of the above]	3.6	3.5

Given $F = kQq/r^2$

2. Two very small spheres A and B have charges on them as indicated in diagram

A 4 metres B
q . ________________________________ . 2q

How does the magnitude of the force exerted on **A** by **B** compare with the magnitude of the force exerted on **B** by **A?** The force exerted on A is

Responses **Percentage**

		1989	1990
A.	four times the force on B	3.7	6.9
B.	two times the force on B	22.8	18.8
*C.	the same as the force on B	14.8	13.1
D.	half of the force on B	49.5	51.3
E.	[did not choose any of the above]	9.2	9.9

3 A 30 Ohm resistor and a 60 Ohm resistor are connected in series to a battery. Compared to the rate at which heat is produced in the 30 Ohm resistor, the rate at which heat is produced in the 60 Ohm resistor is

A.	the same as that produced in the 30 Ohm resistor	12	11.2
*B.	twice as much as that produced in the 30 Ohm resistor	52.3	48.8
C.	half that produced in the 30 Ohm resistor	20	20.4
D.	is zero like that produced in the 30 Ohm resistor	2.5	3.1
E.	[did not choose any of the above]	13.2	16.6

$H = I^2 R t$

4. The resistance of a given length of wire of circular cross section area is equal to R. A second wire is made of the same material, same length but with a diameter two times that of the original wire. The resistance of the second wire is

A.	4R	3.4	8.0
B.	R/2	43.0	38.5
C.	2R	27.4	28.5
*D.	R/4	6.8	5.4
E.	[did not choose any of the above]	19.4	19.3

Given $R = p \frac{\text{length}}{\text{Area of cross section}}$ where p is the resistivity

FURTHER DISCUSSION:

With regard to the first of these examples it is observed that the idiosyncratic tendencies relating to force and the Third Law still appear.

Errors seem to occur from misinterpreting the meaning of a physical representation; in formulating or interpreting equations or just plainly through not an applying a critical mind to it.
In Question 1 the overiding incorrect approach is that in mathematical relationships the functional relationships are always simple ==> starting from direct proportionality.

Halving the distance leads to doubling the force between the charges.
Correct approach would have involved direct substitution into the equation and manipulation.

In Question 2 the same points raised in Q1 are confirmed.
Overidden : the force between the two charges is mutual.
Similar arguments can be used in response to an analysis of the other questions. or example in Question 4 two times the diameter results in half the value for the resistance.

Follow up investigations of student thinking {through personal interviews etc } have indicated that these processes are related to incorrect ideas [conceptions] that students have particularly in relation to the concepts of force, charge and energy.:

that a body possesses a force [force contained inside the body]
is pumped up full with force
is charged up with force [energy]
is charged up [has more charge] as a result of which the body will have relatively more energy [force or charge] with with to repel other bodies of lesser energy.

CONCLUSION

Bearing in mind the observations cited above, and extensive experience in teaching Physics, it seems clear that the teacher of a first year level course should seriously consider formal remediation of the mathematical skills necessary to the study of Physics at the beginning of the Semester.

There will perhaps be many teachers who will be quick to associate the problems cited above with the inadequate preparation of students who come from a disadvantaged school system, and perhaps justifiable so. We would like to highlight a couple of other conclusions that can be made.

(i) Even if to a lesser extent the issues with mathematical manipulation are larger than the associated historical disadvantage. The work done at the University of Houston bears testimony to this assertion [Hudson and Rottman (N = 1403)]. The work of Lochhead mentioned earlier does identify some related issues.

(ii) Other more elaborate investigations indicate that college students' patterns of error in translating from words to equations are the results of partially successful intuitive approaches in writing equations, that the student has, rather than carelessness [Clement J, and others: Focus on Learning Problems in Mathematics Vol 3 (3) July 1981.]

(iii) The work by Yam K Chin points to the fact that even pre service student-teachers with adequate university preparation [maths and physics majors] have difficulties with the concepts related to proportion. One of his findings relates to the lack of precision in the use [/and understanding] of words; and he also suggests that language is an important determining factor.

(iv) The problems are real and affect many of us who admit students into our undergraduate programmes.

There is evidence that the deficiency the student has, in deriving the correct equation, often may stem from the fact that data cannot speak for itself. The person reading the data must reorganise it into a meaningful pattern. Unfortunately, for many of our students the particular patterns used to symbolise the data are rarely stated. It is not unusual for a student to symbolise a pattern incorrectly with an equation that has meaning for him, for example the incorrect 9s=T above. In the case above the equation can actually be used successfully, to for example, predict additional points. Such a student may actually have difficulty in understanding why he is mistaken [when he is told that he is in fact mistaken]. Unless the student "discovers" why his approach does not work he is unlikely to surrender it cognitively.

Our own observations are that often students expect to reach conclusions from insufficient / inadequate information: for example, solving for two unknowns from one equation. In teaching we ought to create novel situations that expose this fallacy. It is likely that by the time a student realises that he cannot solve for two unknowns from one equation more appropriate reasoning flourishes, and his mindset becomes more systematic and analytic.

Certainly at first year level we should place greater emphasis on developing translating skills. Lochhead [1980] makes the point that to accept glibly that mathematical expressions in themselves are a most reliable means of communicating ideas is rather dangerous. There is evidence that the interpretation of mathematical statements may actually be a confusing process; that mathematical manipulation, and notation in general, may actually hinder understanding. Although in themselves they are a most powerful aid in developing understanding; it should not be taken for granted that using them always facilitates understanding.

The hope of university teachers of physics is often that by accepting into courses students who have managed mathematics well at the stage of leaving school there is no need to address further the needs of students; that the insistence that Physics 1 students take mathematics concurrently is enough. Unfortunately we, as physics teachers are the only ones who can assess realistically with any confidence that any lessons have in fact been learned, because we interact with these students at the level where learning is meant to happen. One thing which to our minds is certain is that lecturers should spend more time thinking about [/ assessing] how simple mathematical manipulation may affect the learning of general physics.

References:

Chin, Jap K : Int Journal of Science Education August 1992

Clement J, **Narode** R, and **Rosnick** P : Focus on Learning Problems in Mathematics: vol 3 (3) July 1981

Hudson and **Mc Intire**: American Journal of Physics 45(5) 1977

Lochhead J: Journal of Mathematical Behaviour 3(10) 1980