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Paper Title: Childrens Reasoning about Vector-Problems
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Childrens Reasoning about Vector-Problems

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Problems to define the concept of force and difficulties to integrate vector components are closely related. The well known impetus belief gives an illustrative example: According to this misconception force is interpreted as some kind of energetic substance which can be transmitted and used up. The impetus belief is assumed to be based on daily experience proving that passive objects, set in motion by an external agent (like an object being pushed by a child), slow down and stop after a while apparently on their own (McCloskey, 1983). Frictional forces which cause negative acceleration and compensate the objects forward movement usually are ignored. When physics novices explain such motion problems they rarely mention the existence of more than one force (Clement, 1982; White, 1983). One reason for the development of physical misconceptions like the impetus-belief might thus be the wrong identification or integration of relevant vector components. Empirical evidence for this hypothesis comes from a developmental study dealing with trajectory movements:

Kaiser, Proffitt & McCloskey (1985) observed that preschoolers expect a ball rolling off the edge of a table to fall straight down. The percentage of subjects indicating that the ball continues its forward-movement increased from only 9% for preschool and kindergarten children to 50% for fifth and sixth graders. The authors explain their results with preschoolers inability to think abstractly about the two velocity vectors determining the falling ball's motion. They argue that younger subjects focused their attention on the most salient vector operating on the ball at any given time because they are generally inable to perform mental combinations (Flavell, 1977). Repeated contradictory experience finally leads them to recognize the relevance of both vectors. As the obtained data reveal this change seems to take part during elementary school years. According to Piaget (1978) children are not able to perform vector-addition before they reach formal operational level. It follows that older elementary school-children might recognize the relevance of two different vector-components but not yet be able to integrate them correctly.

By assuming that first the rolling balls impetus and later gravity determines the observed trajectory movement they develop a senseful way to consider two vectors without the need to perform vector-addition.

From a methodological point of view trajectory-movements are no suitable paradigm to study intuitive knowledge about vector integration. Whenever two or more force vectors act on an object for different time intervals, different reasons for malperformance must be taken into consideration: Subjects could draw wrong conclusions because they hold general misconceptions concerning the nature of force. They might also neglect one of the relevant forces determining the objects movement. As a third possibility they might recognize all vectors but have difficulties to integrate information about intensity and direction of several components correctly. By presenting two or more *forces of the same kind*, *produced by visible agents and acting on a target object simultaneously and continously*, experimental tasks designed to investigate intuitive knowledge about vector integration can prevent confounding these aspects. This kind of procedure was chosen by Chollet-Levret (1973). In her paradigm, forces were presented as weights, pulling at three different cords. All cords were tied to an object fixed in the center of a round table.

Chollet-Levret (1973) asked her subjects to determine a third force that could balance out two given forces. Based on data from clinical interviews, she observed that five to seven-year olds showed no consistent response-pattern. False beliefs found among older subjects refered to arguments like: two given forces either compete with each other or they do cooperate. In the former case, children said that one of the given forces (usually the stronger component) would win and pull the object exactly in its direction. In order to prevent this the third force was positioned at the opposite side of the stronger force. In the later case, children ignored size-differences between both given vectors. They always positioned the third force at the opposite side of the bisector of the angle between the two given forces. Only children older than ten years were able to find the proper solution. In line with Piaget Chollet-Levret concluded that children do not integrate force vectors correctly before they reach formal operational level (see also Erickson & Hobbs, 1978). Because of methodological problems typically associated with clinical interview studies these results should be interpreted with caution. Nevertheless they give rise to interesting hypotheses concerning the development of misconceptions about force interaction:

Children who predict the movement of an object being pulled by two forces may react according to a general rule called the *one-force-only-strategy*. Deciding that the object will move in the direction of the stronger force alone, they omit integrating information about the direction of both vectors. Another rule expected to be frequently used by younger subjects is the *bisector-strategy*. Children following this strategy predict that the object will always move in the direction of the bisector of the angle between two given forces. They integrate information about the direction of both forces but neglect information about different sizes of both vectors. These misconceptions should occur more frequently among younger than among older elementary school children and adults. In order to investigate the empirical relevance of both misconceptions, Study 1 was conducted.

Study 1

METHOD

Sample: A total of 96 subjects, 16 adults and 80 elementary school children participated in this study. All adults (mean age: 33;7) had followed the highest German school track (Gymnasium). None of them was physics expert. First, second, third and fourth graders (20 children per group; mean ages: 7;1, 8;1, 9;3 and 10;4 years respectively) had been recruited from an elementary school placed in Marburg, Germany. Males and females were equally devided among each age group.

Material: A modified version of the force-table was constructed (see Figure 2). Instead of three only two forces were acting on the target object. The forces were represented by different numbers of weights positioned on small plates which hang down from two seperate cords pulling at the target object. The later was fixed at the center of a platform surrounded by a wall. This wall could be rotated around the platform. Subjects were instructed to predict where the object would move after being released by the experimenter. Therefore an opening in the wall had to be turned in a position that would allow the object to pass through and fall in a little box behind the opening. In physical terms, two vector forces were acting on the object simultanously and continously. Subjects had to predict the direction of the resultant. The number of weights on both plates provided proportional information about the size of the force vectors and their direction was determined by the angle between the strings.

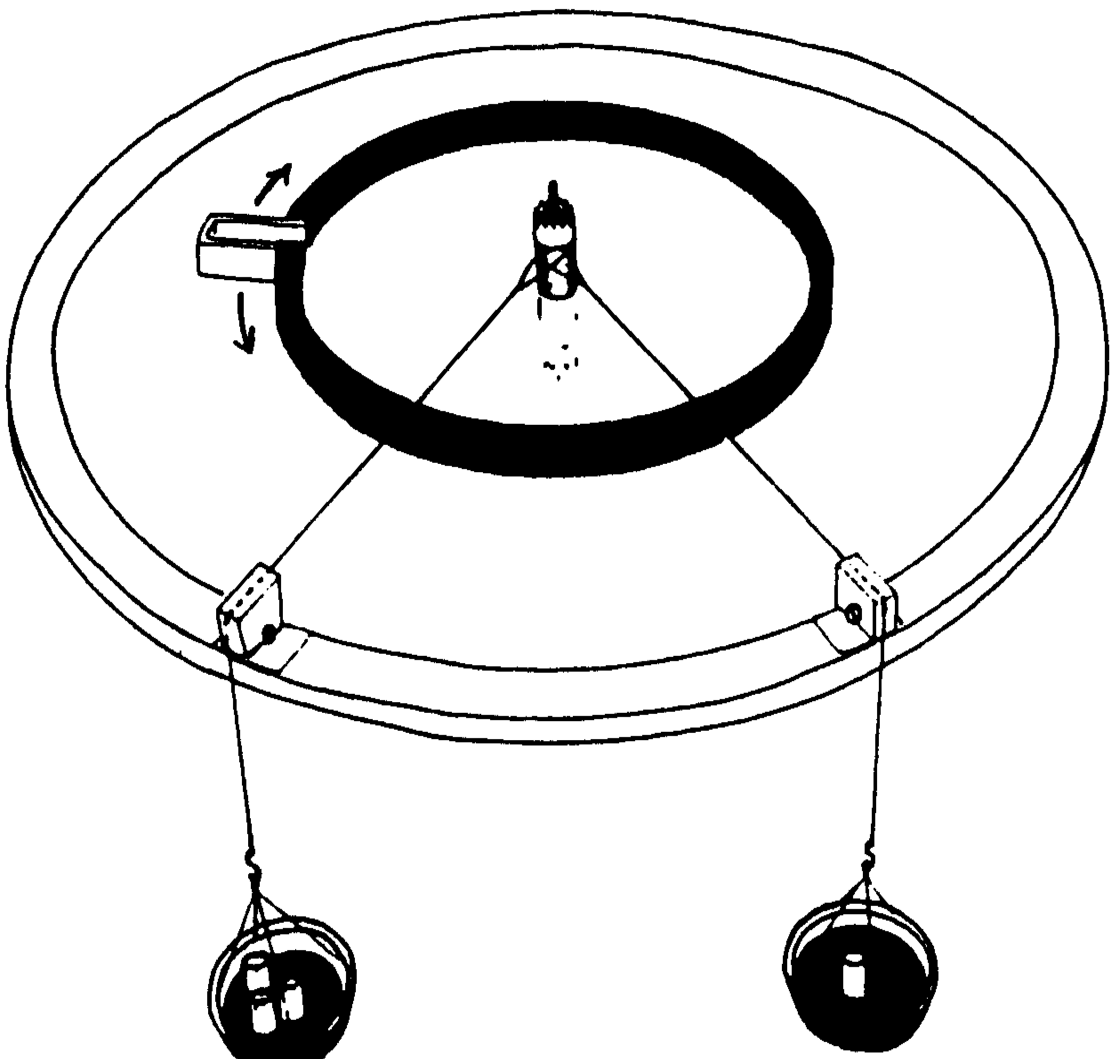

Figure 1: Modified force table (Weights-Pulling)

Procedure: Children and adults were all tested individually. Following three pretrials, subjects were confronted with nine factorial combinations of three proportions of forces (1:2; 1:3, 1:6) and three angles (45°; 75°; 105°) presented in random order. After these regular vector problems, three special cases were presented (A: angle 180°, proportion of weights 1:6; B: angle 180°, proportion of weights 1:1; C: angle 75°, proportion of weights 1:1).

The task was embedded in a story of a King (target object), who got tired skating on a frozen lake (platform) and wanted to be pulled into his royal bed standing at the shore (box behind the opening in the wall). Whereas the experimenter determined the direction and intensity of the pulling forces, the subject was asked to turn the wall by bringing the royal bed in a position that would allow the King to slide in.

No feedback was given during the entire experimental session. Subjects were encouraged to think aloud while working on the task. Their verbal comments were tape-recorded and transcribed. Following the session the experimenter asked subjects how they had come to their predictions and if this game reminded them of anything familiar.

RESULTS

Coding schema for regular vector problems: The raw data were transformed into typical response categories: If a person consistently predicted, the object would either move to the strong or to the weak side in eight out of nine trials, her answers were characteristized as identifying the *one-force-only strategy*. If all of the subjects answers fell into the centered third part of the angle this pattern was identified as the *bisector-strategy*. If a person positioned the opening somewhere between the bisector of the angle and the stronger force eight or nine times her response-pattern was said to follow the *two-forces-integrated strategy*. People answering this way can be supposed to have basic knowledge about vector addition: They know that the resultant of both vectors points in a direction none of the two visible forces does. They are also aware of the fact that the stronger force has more influence on the movement direction of the target object than the weaker one. In a pilot-study conducted with 40 subjects of different ages I observed that several subjects were undecided whether the weaker of both forces would have any influence on the objects movement. Their answers vacillated between the one-force-only- and the two-forces-integrated strategy. These subjects appeared to be in a transitional stage, already doubting that the one-force-only strategy is correct but not yet being sure about the correctness of the two-forces-integrated strategy. Their strategy was labled *two-forces-considered*. People who didn´t fit into any of these categories were summed up in a group called *rest*.

Strategies used for solving regular vector problems: Figure 2 reports the percentage of subjects using the one-force-only-, the bisector-, the two-forces-considered- and the two-forces-integrated strategy in each age-group. As can be seen, the responses of most subjects fell into one of the interesting categories. The large majoritiy of first, second, and third graders (80%, 80% and 85% respectively) predicted that the object would move straight in the direction of one of the forces (usually the stronger one), whereas less than half of the fourth graders (45%) and only a few adults (12.5%) showed this response-behavior. Only very few first and second graders (10% and 5% respectively) thought, the object would always move in the direction of the bisector of the angle. This response-pattern was not found among children older than eight years. The two-forces-considered strategy could most frequently be observed among fourth graders (35%). All of them vacillated between integrating both forces and predicting that the stronger force alone would determine the objects movement. The same was true for adults following this strategy (12.5%). None of the first graders, very few of the other elementary school children (5% to 10%) and more than half of the adults (62,5%) consistently integrated both forces. The percentage of wrong solution attempts (rest, one-force-only- and bisector-strategy taken together) significantly decreased with age (Chi2 = 30.45, df = 4, $p < .05$).

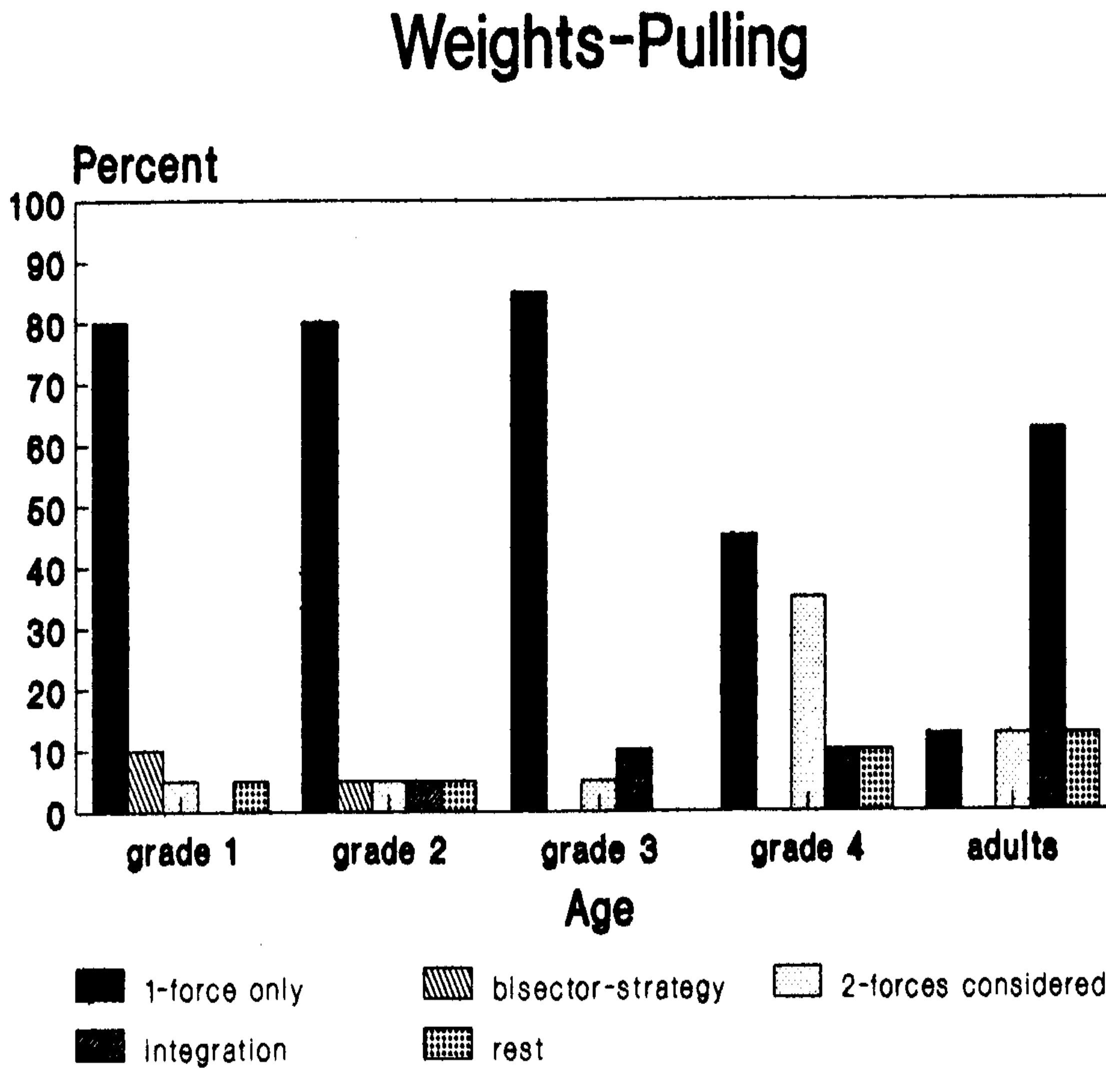

Figure 2: Strategies used for regular vector-problems in different age-groups (Weights-Pulling)

Strategies used for special vector problems: When both forces were positioned at the opposite side of each other (angle 180^{o}, weight proportion 1:6 or 1:1), the majority of all subjects in each age-group knew the correct solution. When both forces were of equal size (1:1) but the angle was smaller (75^{o}), this percentage was slightly lower (see Table 1).

Table 1: Percentage of correct solutions for special vector problems (Weights-Pulling)

	age-group				
	1st grade	2nd grade	3rd grade	4th grade	adults
180° 1:6	65	85	85	100	100
180° 1:1	85	90	95	90	100
75° 1:1	70	50	75	85	100

Protocol data: Think aloud protocols revealed that first, second, and third graders using the one-force-only-strategy based their predictions on the argument that the difference in number of weights on both plates determines to which side the object will move. Very often children explained that the plate with more weights on it would move down whereas the other plate would move up. (In physical reality both plates move down in most of the presented cases.) Some fourth graders used arguments typical for younger age groups, others unsuccessfully tried to consider the role of the angle. Children who were able to overcome this misconception frequently specified, under which circumstances the one-force-only- or the bisector-strategy lead to correct answers. Thought experiments of this kind played an important role in children as well as in adult reasoning. Many children called the aparatus a balance scale; they compared the weights on the two plates with the two scales of a balance-beam.

DISCUSSION

Whereas the majority of adults integrated both forces correctly, a strong misconception predominated up to the age of about 10 years: Most children believed that the movement was exclusively determined by the stronger force. Some younger children expected the object to move in the direction of the bisector of the angle. In accordance with the results obtained by Chollet-Levret (1978) the one-force-only- and the bisector-strategy could thus be observed among elementary school children. In contrast to her observations, first graders did not answer inconsistently but showed the same response-pattern as the older children. This may be a consequence of making the task easier by presenting two instead of three forces. Among the fourth graders many children proved to be in some kind of transitional stage: They could not decide whether the object would move straight in the direction of the stronger force or somewhere between the bisector of the angle and the stronger side.

Children solved special vector-problems much earlier than others. Considering the fact that these problems are less complex, this finding makes sense: Whenever both forces are of equal size, the object will move in the direction of the bisector of the angle. This solution does neither change with the absolute size of both forces nore with the size of the angle they include (except opposing forces). Whenever both forces are opposing each other, the object will move in the direction of the stronger force (except for a size proportion of 1:1). Given equally sized forces, the object will not move at all.

Despite the fact that the presented data seem to confirm the main findings of earlier explorative studies concerning typical misconceptions and general age trends in childrens performance on the force-table task I hesitate to draw general conclusions about the development of understanding problems of force interaction. The analysis of think aloud protocols has shown that the force-table reminded many children of a balance-scale situation. The usage of this balance scale analogy might explain why only very few children used the bisector-strategy.

Subjects could use different rules when forces are no longer presented as pulling weights on two plates. Support for this hypothesis comes from expert-novice studies on problem solving in physics. They show that novices typically represent any given physics problem by remembering a prototypical situation they judge to be comparable to the given problem (Yates, Bessman, Sly & Wendleboe, 1988). Which situation they remember is influenced by the presence of objects important for the situational context. Considering these findings I expect that presenting the force-table task in a different situational context would change the fequency distribution for the one-force-only-, the bisector-, the two-forces-considered- and the two-forces-integrated-strategy. To test this hypothesis, Study 2 was conducted:

Study 2

METHOD

Subjects: A total of 80 elementary school-children (40 boys, 40 girls), equally devided among first, second, third and fourth graders and 15 adults (8 males, 7 females) took part in Experiment 2. Their mean ages were 6;8, 7;5, 8;8, 9;6 and 27;6 years respectively. Children had been recruited from an elementary school placed in Marburg, Germany. All adults had followed the highest German school track (Gymnasium).

Experimental task: The same modified version of the force-table was used as in Experiment 1: Two forces were acting on a target object fixed at the center of a platform. Instead of a high wall, a flat ring surrounded the platform. The opening of the ring was covered by a removable carpet. The forces were represented by different numbers of toy-people pulling at sticks connected to the target object (see Figure 3). Subjects were told that two groups of cowboys wanted to pull a ton standing in the middle of a frozen lake to the shore.

Aparatus

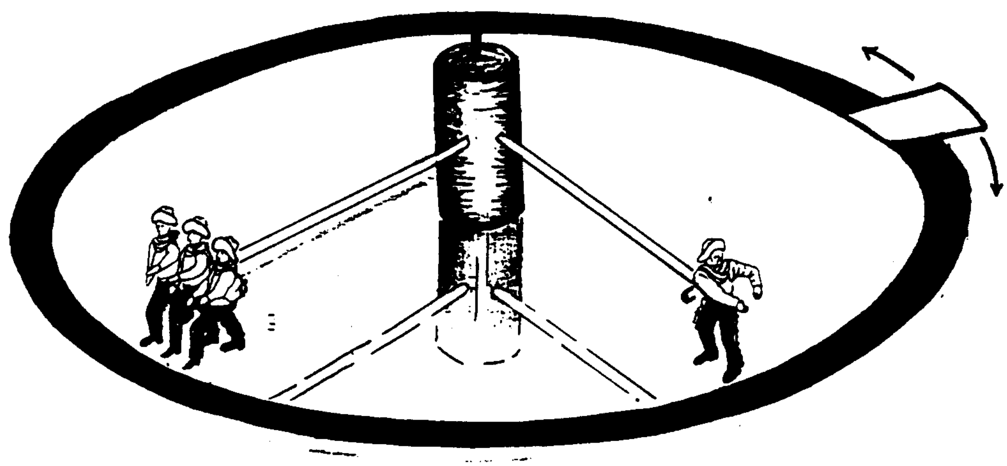

Figure 3: Modified force table (People-Pulling)

The direction and intensity of both pulling forces were determined by the experimenter. She asked the subjects to predict in which direction the ton would move. By turning the ring subjects had to bring the carpet in a position that would allow the ton to be savely transported to the shore. The experimental procedure and data analysis were identical to Study 1.

RESULTS

Strategies used for solving regular vector-problems: As can be seen from Figure 4 the great majority of subjects in each age-group used one of the defined strategies. Almost all first and second graders showed systematic misconceptions. About the same number of children in both age-groups prefered the one-force-only- and the bisector-strategy. Third and fourth graders showed the greatest variance in their response-patterns: Each of the identified rules was used by some subjects and none of them clearly predominated. Adults always chose the two-forces-considered- or the two-forces-integrated-strategy. When subjects using the two-forces-considered- and the two-forces-integrated-strategy were summed up in each age-group and contrasted with those showing completely incorrect response-patterns (rest, one-force-only- and bisector-strategy) a significant decrease in wrong solution attempts was observed ($Chi2 = 52.56, df = 4, p < .05$).

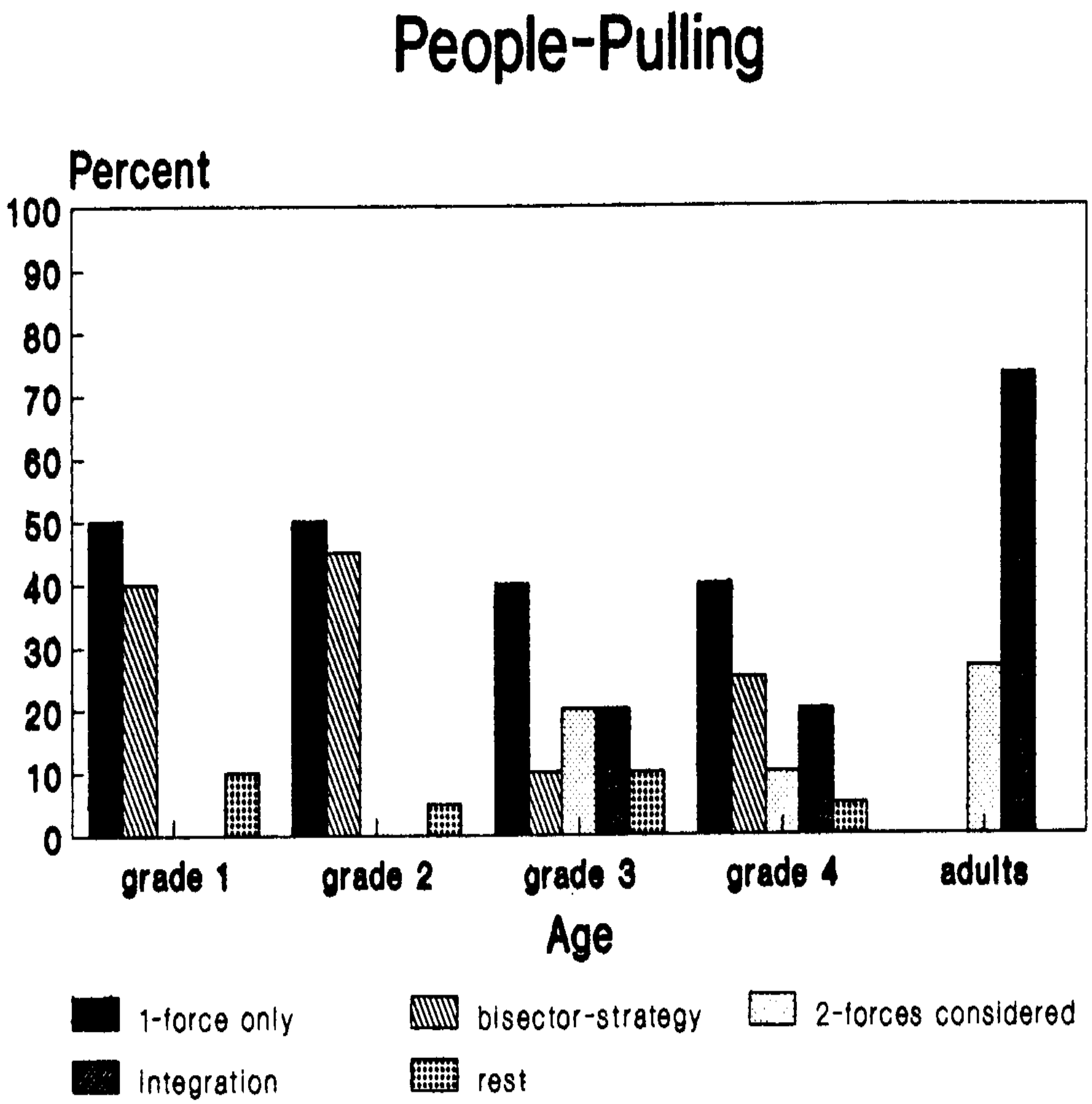

Figure 4: Strategies used for regular vector problems in different age-groups (People-Pulling)

Strategies used for solving special vector-problems: Concerning the three special vector-problems presented at the end of the session between 50% and 65% of the first and second graders knew the correct solution. Among the third and fourth graders this percentage varied between 75% and 100%. Adults never gave wrong answers (see Table 2).

Table 2: Percentage of correct solutions for special vector-problems (People-Pulling)

	age-group				
	1st grade	2nd grade	3rd grade	4th grade	adults
180^o 1:6	65	85	85	100	100
180^o 1:1	85	90	95	90	100
75^o 1:1	70	50	75	85	100

Protocol data: Children explained their predictions with arguments different from those used by subjects who took part in the weights-pulling experiment: If they thought the ton would move exactly in the direction of the stronger force they typically interpreted the cowboy-pulling situation as a social conflict where the stronger party succeeds in putting through its intention. Subjects believing the ton would always move in the direction of the bisector of the angle frequently argued, the ton could slide neither to one or the other side since it was being pulled by two different groups simultaneously. Again the situation was interpreted as a social conflict. In this case, a compromise was considered to be the proper solution. Nobody ever mentioned the comparison with a balance-scale. Instead many children reported concrete experiences with situations in which they had tried to pull an object together with other people.

COMPARISON BETWEEN THE RESULTS OF STUDY 1 AND 2

Independently of the situational context, a significant decrease in wrong solution attempts could be observed. Both versions of the force-table-task lead young children to use either the one-force-only- or the bisector-strategy. Which of these strategies they prefered depended on contextual variables associated with the task. Table 3 shows the percentage of subjects choosing each of the identified strategies, summed up over all age-groups under the condition Weights-pulling and People-Pulling. The frequency distributions for Experiment 1 and 2 differed significantly (Chi2 = 22.98, df = 4, $p < .05$).

Table 3: Percent of strategy use in Study 1 and Study 2: Weights-pulling and People-pulling

	one-force only	bisector-strategy	two-forces considered	two-forces integrated	rest
Weigths-pulling	62.5	3.1	12.5	15.6	6.3
People-pulling	37.9	25.3	10.5	20.0	6.3

GENERAL DISCUSSION

When children and adults were asked to predict the movement direction of an object being pulled by two forces, the frequency of wrong answers significantly decreased with age. In contrast to findings of prior studies (Chollet-Levret, 1978; Kaiser, Proffitt & McCloskey, 1985) even six year-olds based their judgement on information about both presented vectors. Nevertheless, elementary school children of all age-groups typically used one of two misconceptions: the one-force-only- or the bisector-strategy. According to Piagets theory this behavior could be explained by general limits of cognitive information processing in young children. Instead of integrating information about the size and the spacial orientation of two vectors, subjects who have not yet reached formal operational level only consider one of both relevant vector dimensions. When focussing the size dimension, they believe, the object will always move in the direction of the stronger force. When focussing the spatial orientation, they predict that the object will always move in the direction of the bisector of the angle enclosed by the presented forces.

Based on the results of our experiments, I suggest another explanation for the development of the one-force-only- and the bisector-strategy. As both reported studies show, special vector problems like those with opposing or equally sized forces get solved earlier than others. Proper solutions for special vector problems are identical with the one-force-only- and the bisector-strategy respectively: Whenever both forces are opposing each other, the stronger force alone determines the movement direction of the target object (except for equally sized forces). Whenever both forces are of the same size, the object will move in the direction of the bisector of the angle (except for opposing forces). In daily experience children are frequently dealing with special vector-problems of this kind. These experiences might lead them to develop the one-force-only- and the bisector-strategy as heuristic rules to predict the movement behavior of any target object being pulled by two forces. Since each of both strategies represents a basic postulate of vector addition, a generalisation of these rules to regular vector problems makes sense: The one-force-only-strategy correctly assumes that the stronger of two forces has a greater influence on the objects movement than the weaker force. The bisector-strategy points out that none of both forces alone determines the objects movement behavior.

The one-force-only- and the bisector-strategy, identified as misconception when being applied to all kinds of (two force) vector problems, might thus go back to a false generalisation of solution principles developed in dealing with special vector-problems. Additional evidence for this hypothesis comes from subjects verbal comments. They show that discovering the integration-rule was often proceded by the insight that neither the one-force-only- nore the bisector-strategy lead to a correct solution for all presented vector problems. Thought experiments motivated subjects to specify conditions, under which one or the other strategy was the right one. This specification process induced a cognitive conflict between both availiable solution principles. By solving this conflict, some older children and adults developed the integration rule. In contrast to Piagets explanation our observations underline the relevance of subjects experience with special vector problems for the development and later abandonment of false beliefs concerning regular problems of force interaction.

Further arguments stressing the importance of concrete experiences in childrens reasoning on interaction of forces come from a comparison of the data obtained in Study 1 and Study 2. It was found that the presentation of different force-symbols lead to diverging frequencies for each of the identified misconceptions. When weigths were pulling, the one-force-only-strategy clearly dominated. When people were pulling, the bisector strategy was used at the same rate. These differences were larger among younger than among older subjects. Again verbal protocols gave hints for a detailed interpration: Young children confronted with the Weights-pulling-task based their reasoning on an analogy between the force table and a balance beam. This false analogy made them believe that the heavy weight always moves down - pulling the target object in its direction. Subjects taking part in the People-pulling-Study solved the problem by remembering a situation in which they had to pull an object or observed other people doing so. Reasoning on interaction of forces can thus be influenced by the way vectors are represented. Children do not use a general problem solving schema for diverging force problems. Instead they try to remember situations comparable to the given context and apply rules they have already learned from experiences with these situations. In this way their problem solving behavior can be interpreted as being highly adaptive (Anderson & Wilkening, 1991).

What situations are judged to be comparable depends on objects presented in the task. Older children (fourth graders) and adults learned to ignore unimportant context-variables whereas younger children showed strong tendencies to interpret the presented force-problem on the basis of concrete experiences and objects associated with the given task. This developmental trend parallels the well documented novice-expert-shift for students concerning problem solving in physics. Intuitive knowledge about the basic principles of vector addition seem to be a function of experiences with different situational contexts representing the same physical problem

(Larkin 1983; 1985). Young children who usually lack this experience errorneously tend to draw back on knowledge about special vector problems or about superficially comparable situations when solving a given experimental task. More experienced subjects have learned to differentiate which rules are correct for what kind of situations. Based on their increased knowledge-system they are able to develop the integration-strategy .

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