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Narrowing the Gap Between Doing Mathematics In and Out of School: Suggestions for Teaching

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Research in the last fifteen years has indicated a burgeoning interest in examining the mathematics practice in distinct cultures (e.g., Brenner, 1985; Lancy, 1983; Saxe, 1991) and everyday situations within cultures (e.g., Carraher, Carraher & Schliemann, 1985; de la Rocha, 1985; Harris, 1987; Lave, 1988). Research on mathematics practice in distinct cultures has tended to look at the mathematics practice of a whole culture, whereas research on mathematics practice in everyday situations within cultures has focused on one situation or work context (e.g., grocery shopping, carpentry) within a culture. Some of this research (e.g., Carraher, Carraher & Schliemann, 1985) has contrasted mathematics practice in school with mathematics practice in everyday situations and noted the gap between these two.

Knowledge gained in out-of-school situations often develops out of activities which: (a) occur in a familiar setting, (b) are dilemma driven, (c) are goal directed, (d) use the learner's own natural language, and (e) often occur in an apprenticeship situation allowing for observation of the skill and thinking involved in expert performance (Lester, 1989). Knowledge acquired in school all too often grows out of a transmission paradigm of instruction and is largely devoid of meaning (lack of context, relevance, specific goal). Resnick (1989) has argued that schools place too much emphasis on the transmission of syntax (procedures) rather than on the teaching of semantics (meaning) and this "discourages children from bringing their intuitions to bear on school learning tasks" (p. 166).

Students need in-school mathematical experiences to build on and formalize their mathematical knowledge gained in out-of-school situations. An important part of a mathematical experience in school is the guidance and structure that can be provided by a teacher to help students make connections among mathematical ideas. Some researchers (e.g., Lave, Smith & Butler, 1989; Schoenfeld, 1989) have suggested that teachers should establish master - apprentice relationships with their students to help initiate them into the mathematics community (persons who are learners and doers of mathematics). Working with others toward common goals, being actively involved in doing mathematics, and discussing and refining mathematical ideas are all ways of being a part of the mathematics community. At present, however, many students become isolated from the mathematics community rather than becoming a part of it. A key reason for this isolation is the wide gap that exists between mathematics practice in school and in out-of-school situations.

My interest lies in closing the gap between doing mathematics in school situations and doing mathematics in out-of-school situations. This article discusses suggestions for the learning and teaching of mathematics stemming from several studies examining mathematics practice in everyday situations. The studies reported here were based on the assumptions that mathematical knowledge is a type of cultural knowledge and as such, the mathematics curriculum associated with formal schooling should engage students in mathematical practice where the knowledge is situated in the context of its use.

Through participant observation in contexts involving carpet laying (Masingila, 1992b), interior designing, and retailing, data were collected about the mathematics concepts and processes used in these contexts, as well as learning through apprenticeship in the carpet laying context. Analysis of mathematics curricula and student problem-solving efforts involving problems from these contexts afforded a comparison of doing mathematics in and out of school. Various conceptual, theoretical, and methodological frameworks guided the conceptualization, design, and conduct of these studies: a cultural framework of ethnomathematics, an epistemological framework of constructivism, a cognitive framework of activity theory, and a methodological framework of ethnography. (For a more detailed discussion of these frameworks, see Masingila, 1992a, 1993.)

The following list of conclusions briefly summarizes my findings from this research:

1. Carpet layers, interior designers, and retailers engage in doing mathematics. That is, mathematics concepts are present in their work and they use mathematical processes as they solve problems they encounter in doing their jobs. Job situations are problematic because of the numerous constraints inherent in floor covering, interior design, and retailing work. For example, in carpet laying: (a) floor covering materials come in specified sizes (e.g., most carpet is 12' wide, most tile is 1' x 1'), (b) carpet in a room (and often throughout a building) must have the nap (the dense, fuzzy surface on carpet formed by fibers from the underlying material) running in the same direction, (c) consideration of seam placement is very important because of traffic patterns and the type of carpet being installed, and (d) tile must be laid to be lengthwise and

widthwise symmetrical about the center of the room. Interior design work has the same type of constraints as carpet laying—size of materials, patterns and seam considerations, physical structures in a building, symmetry of ceiling grids—while in retailing one has to consider factors such as markups, markdowns, shrinkage (negative difference between the final book inventory

and the actual verified physical inventory), how the time of year affects sales,

and operating expenses.

2. Apprenticeship serves as a good method of learning and teaching in the carpet laying context.

3. Textbooks often do not provide constraint-filled situations to engage students in problem solving.

4. Students often have a narrow understanding of mathematics concepts, such as area, and a limited range of problem-solving skills and strategies perhaps because they have not been exposed to constraint-filled problems that engage them in problem solving.

SUGGESTIONS FROM RESEARCH

In reflecting on these studies and other research examining mathematics practice in everyday situations, I have formed some suggestions for the school mathematics curriculum and the methods used to teach school mathematics.

For the School Mathematics Curriculum

In comparing the school-based knowledge of students with the experienced-based knowledge of workers in these contexts I found that some of the difficulty the students had in solving problems from these contexts was related to the students' lack of exposure to rich, constraint-filled problems.

The main suggestion for the school mathematics curriculum stemming from research on mathematics practice in everyday situations is that the curriculum should include a wide variety of rich problems that: (a) build upon the mathematical understandings students have from their everyday experiences, and (b) engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations.

At present, students are not encouraged to and maybe even discouraged from making connections between how they do mathematics in school and how they do mathematics out of school. Students often do not see school mathematics as connected to the real world, and as a result they often do not evaluate solutions to school exercises to see if they make sense. The following exercise, taken from the Third National Assessment of Educational Progress (Carpenter, Lindquist, Matthews & Silver, 1983), illustrates the fact that students often do not try to make sense of school mathematics:

- An army bus holds 36 soldiers. If 1128 solders are being bussed to their training site, how many buses are needed? (p. 656)

Of the students who worked this exercise only 23% gave the correct answer of 32 buses, while 29% indicated that "31 remainder 12" was the number of buses needed. These students would never give this answer if they were thinking of this exercise in its real-world context. Instead, these students saw this exercise in the context of a school situation and thus they did not evaluate the reasonableness of their answer.

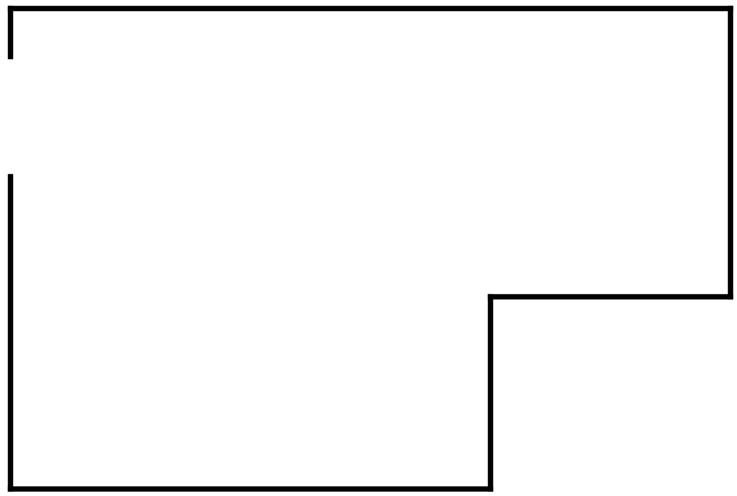
Often students do try to bring their out-of-school experiences to bear upon work in school when the work engages them in problem solving. For example, several of the students I worked with used personal experiences to give them insights into solving the problems. However, the present curriculum is frequently so divorced from students' out-of-school experiences that it is very difficult for students to connect the out-of-school world and the in-school world. Thus, doing mathematics in school often has little meaning for students.

Besides having problems that build upon the students' everyday mathematical experiences, the school mathematics curriculum should engage students in doing mathematics by providing them with rich, constraint-filled problems. Schoenfeld (1987) has suggested that what is needed in mathematics classrooms is to "engender a culture of schooling that reflects the use of mathematical knowledge outside the school context" (p. 214). Consider the following exercise taken from a pre-algebra book:

- Find the cost of carpeting a floor that is 15 feet by 12 feet at $24.95 per square yard. (Shulte & Peterson, 1986, p. 527)

Although this problem uses an everyday context, the absence of actual constraints makes the problem artificial; the textbook example is simply a computational exercise placed in the context of an everyday situation. The effort required by a student to find an answer to this exercise does not reflect the way mathematical knowledge is used in carpet laying. Situations in a carpet laying context involve using measurement concepts in problem-solving situations compounded by real-life constraints.

Instead of stating the exercise as it is above, this example could be made into a problem-solving situation by changing it to the following:



1. Find the measurements of this room. The diagram is drawn in a scale of 1/4 inch = 1 foot.
2. Find the most cost-efficient way to carpet this room given the following constraints:
   - carpet pieces are 12 feet wide
   - the nap of different carpet pieces must all run in the same direction
   - seams should be placed out of normal traffic patterns

Another problem, this one from the interior design context, also engages students in problem solving.

- You need to wallpaper a wall that is 96" high and 10' long. Find the most costefficient way to paper this wall given the following constraints:
   - the wallpaper is 20.5" wide
   - a single roll of wallpaper is 5.5 yds in length
   - the wallpaper has a pattern that repeats every 9" and this pattern must be matched by adjacent pieces

Not only do the above problems have real-world constraints, but they also engage students in the processes of measuring and problem solving. The effort required by a student to find solutions to these problems does reflect the way mathematical knowledge is used in the everyday situations. Problems of this type engage students in mathematics practice and encourage mathematical understanding of the concepts involved.

Besides the suggestion that the school mathematics curriculum should include rich problems that build upon the out-of-school mathematical understandings of the students and engage them in mathematics practice, this study has some suggestions for the teaching of school mathematics.

For Teaching School Mathematics

The above discussion concerning changes in the school mathematics curriculum is all for naught if far more important changes do not occur in the chain of events that affects the way students learn mathematics in school. The research into the mathematics practice in carpet laying, interior design, and retatiling and the way the students approached problems from these contexts suggests three key ideas for teaching school mathematics.

The first is that teachers should build upon the mathematical knowledge that students' bring to school from their out-of-school situations. As discussed above, students try to make connections between the in-school and out-of-school worlds, but the mathematics practice in school often discourages or prevents such connections.

The second suggestion arises out of the way in which floor covering helpers learn about installation work and from the obvious lack of understanding of some concepts, such as area, that the students demonstrated: Teachers should introduce mathematical ideas through situations that engage students in problem solving.

The third suggestion comes directly from the master - apprentice relationship that enables floor covering helpers to become expert carpet layers: Teachers should establish master - apprentice relationships with their students to guide students in doing mathematics and help initiate them into the mathematics community. Note that while the apprenticeship model used in work situations is a viable model for those contexts, the model needs to be adapted for use in the classroom. This adaptation will be discussed later in the article.

Build on students' out-of-school knowledge. All students bring to school mathe-matical knowledge from everyday situations they have experienced. This knowledge is often hidden and unused by the students in school as they learn to use the mathematical procedures that the teacher demonstrates and evaluates.

Just as the mathematics practice of everyday activity is ignored by teachers in school, mathematics practice in schools is likewise devalued by students because of their lack of use of it in out-of-school situations: "There exists no legitimate field of practice other than the classroom itself" (Lave, Smith & Butler, 1989, p. 74). As D'Ambrosio (1985) has noted, people in out-of-school situations often use very different mathematical procedures and thinking processes than those taught in school.

However, if teachers engage their students in conversation, listen to them, and encourage and observe their informal methods of solving problems, much can be learned about the out-of-school situations their students have experienced. Lester (1989) offered several suggestions for helping students make connections between the in-school and out-of-school world. One suggestion was to have students create their own problems. Another suggestion was to encourage "students to solve problems in more than one way and to share their approaches with each other. By sharing approaches, students will learn about methods used by their peers and also that it is acceptable to use the informal methods they have developed on their own" (p. 34). A third suggestion involves engaging students in mathematics projects that encourage the use of mathematical methods and reasoning.

By building upon the mathematical knowledge students' bring to school from their everyday experiences, teachers can encourage students to: (a) make connections between these two worlds in a manner that will help formalize the students' informal mathematical knowledge, and (b) learn mathematics in a more meaningful, relevant way. "Mathematics teaching can be more effective and will yield more equal opportunities, provided it starts from and feeds on the cultural knowledge or cognitive background" of the students (Pinxten, 1989, p. 28).

Teach via problem solving. Although each of the students I engaged in solving some floor covering problems had been taught about area, none of them were able to use their knowledge of area to solve the following problem:

- Suppose you need a piece of carpet 12 feet by 9 feet. How many square yards should you order from the carpet supplier?

The students were able to change feet into yards. But when they were asked how many square yards were needed, they either found the perimeter or squared each dimension. When I asked what it meant to find square yards, none of the students provided an answer. When I finally identified the task as finding the area of the carpet piece, the students typically replied, "Oh, length times width."

To these students, area is a formula dependent upon the geometric shape of the object. Their understanding of the concept of area is narrow and strongly tied to algorithms for finding area. Because the students have not experienced finding area in a real-life way (at least not in school), they do not have an understanding of area that can be applied to concrete situations. On the other hand, the carpet layers work with area in concrete ways every day and are able to apply their knowledge of area to a wide variety of floor covering situations.

If the students had been involved in solving problems where they encountered dilemmas that required them to find areas, their understanding of the concept of area would have been constructed out of their mathematics practice. This is the goal of teaching mathematics via problem solving. In teaching via problem solving, "problems are valued not only as a purpose for learning mathematics, but also as a primary means of doing so. The teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems" (Schroeder & Lester, 1989, p. 33).

This approach embodies three key recommendations of the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics (NCTM), 1989): (a) mathematics instruction should be carried out in an inquiry-oriented, problem-solving atmosphere, (b) mathematics concepts and skills should be learned in the context of solving problems, and (c) the development of higher-level thinking processes should be fostered through problem-solving experiences.

Teaching via problem solving deviates from the traditional instructional approach of the teacher presenting information (facts) and then assigning exercises in which students practice and apply this information. Using a teaching via problem solving approach means that the mathematical information arises out of the problem-solving activity, along with an understanding of the mathematical concepts and processes involved.

Teaching via problem solving is also consistent with the way in which apprentice workers in the floor covering context learn about estimating and installing. The apprentices are engaged in problem solving every day and gain mathematical knowledge (although they may not be aware of this) through solving these problems. An important aspect of this problem solving is that it occurs in the context of a master - apprentice relationship.

Use the apprenticeship model in the classroom. The floor covering helpers I observed became expert carpet layers by observing the installation process, questioning the installer, participating in the installation process, learning from their mistakes, and coming to know what the installer knows. The installers contribute to this process by maintaining control of the installation process, giving the helpers the opportunity to develop a feel for installation work, and determining the progress of the helper.

A number of researchers have discussed apprenticeship and its application to the classroom (Collins, Brown & Newman, 1987; Lave, 1977, 1988; Lave, Smith & Butler, 1989; Rogoff, 1990; Schoenfeld, 1987, 1989) and have found the apprenticeship model to be a viable one for teaching and learning. However, the apprenticeship model that could be used in a classroom is different in two important ways from the apprenticeship model used in work situations, and in particular in the carpet laying context.

The first difference involves the master - apprentice relationship: In the workplace, a master and apprentice are working one-on-one; in the classroom, a teacher and possibly 30 or more students are working together. In the workplace, the apprentice is guided and directed by the master as he or she participates in the work activity; in the classroom, the students are guided by the teacher, but more importantly are guided and challenged by other students as they work cooperatively in doing mathematics. Thus, applying the apprenticeship model to the classroom implies a heavy reliance on cooperative learning: The teacher creates a classroom environment where students (apprentices) work with other students (apprentices) and in this way the teacher helps initiate the students into the mathematics community.

A second difference between the use of the apprenticeship model in the workplace and in the mathematics classroom is that apprentices in the workplace are constructing situation-specific knowledge; in the mathematics classroom students are constructing mathematics content and processes that are more general, and hopefully can be applied to a variety of situations. The end goal of my suggestion that the school mathematics curriculum should contain rich, constraint-filled problems (e.g., problems from a carpet laying context) is not that students acquire the knowledge necessary to become expert carpet layers. Rather, problems of this type are vehicles for engaging students in doing mathematics and aiding them in developing the mathematical reasoning and problem-solving abilities used by expert problem solvers.

There are three key reasons for using the apprenticeship model in the mathematics classroom: (a) an apprenticeship model enables mathematical knowledge to be developed within a context, (b) cognitive development can occur as students work cooperatively with their teacher, and (c) a mathematics culture is developed within the classroom and students are initiated into this mathematics community.

Using the apprenticeship model, mathematical knowledge is developed within a context and is framed by that context: A "general advantage of learning-as-apprenticeship is that it assumes that knowing, thinking, and indeed, problem-solving activity, are generated in practice, in situations whose specific characteristics are part of the practice as it unfolds" (Lave, Smith & Butler, 1989, p. 64). The apprenticeship model "implies continuity between ways of learning and thinking in school and nonschool settings: Learning is learning, mathematical thinking is mathematical thinking, in and out of school" (Scribner & Stevens, 1989, p. 1).

Vygotsky's concept of the zone of proximal development included the proposal that cognitive processes occur first on the social plane. The individual plane is then formed as these shared processes are internalized and transformed. Thus, the zone of proximal development is a dynamic region of sensitivity to learning the skills of culture, and children develop in this region through participation in problem-solving activities with more experienced members of the culture (Rogoff, 1990; Vygotsky, 1978).

Not only do children develop mathematical knowledge by working closely with more experienced members of the culture, but they themselves become part of the mathematics community. Using an apprenticeship model in the classroom provides a way for initiation into the mathematics community. Schoenfeld (1987) has noted that whereas schools traditionally provide "training in ready-made, prepackaged mathematical procedures . . . apprenticeship does more than provide training: it provides an initiation into a culture" (p. 204).

Thus, using the apprenticeship model in the mathematics classroom allows students to do mathematics in a natural way and be active learners in a "community of people who support, challenge, and guide novices as they increasingly participate in skilled, valued sociocultural activity" (Rogoff, 1990, p. 39).

SUMMARY

In this paper I have offered suggestions related to the school mathematics curriculum, and how school mathematics is taught. Overriding all of these suggestions is that in order for students to learn mathematics meaningfully, the gap between doing mathematics in school and doing mathematics in out-of-school situations must be reduced.

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