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PERSISTENT INCONSISTENCIES IN TEXTBOOK MECHANICS

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ABSTRACT

If concept development is inhibited by inconsistency in what is presented to learners, then introductory mechanics is open to criticism. For example, pressure is used to refer to four conceptually distinct phenomena, scalar tension and vector force are routinely combined, and Newton's third law is regularly mis-applied in three classes of interaction. Means for achieving consistency are suggested.

I believe that instruction for meaningful learning must obey two rules:

1. Conceptually distinct phenomena should never be given the same name: conceptually unique phenomena should always be given the same name.

2. Something which is difficult to learn should not be transformed to something which is easy to learn but which is not valid.

The examples which follow mainly illustrate breaches of the first rule in treatments of *force, net force, tension* and *pressure.*

PROBLEMS WITH THE CONCEPT OF FORCE

The concept of force is central to introductory mechanics and an understanding of mechanics is crucial to Physics, engineering and other sciences, so it is important that students should develop clear, usable concepts. This is not the case at present. A wealth of recent research into the ideas about force and related concepts held by students at school and at university demonstrates that wide discrepancies exist between what they have been taught and the ways in which they think about everyday phenomena.

Apart from any built-in barriers to understanding such as the need to formulate ideas mathematically there appear to be at least four reasons for the failure of conventional instruction to give rise to acceptable and transferable concepts:

> What we perceive through physical sensation and describe as force in everyday language often conflicts with scientific meanings and definitions

This is very unfortunate, for everyday ideas are developed early in life and reinforced by vast amounts of experience and talk. Well known examples include the deep-seated association of motion with force in the direction of motion, and the apparent forces experienced by passengers in accelerating vehicles. Less well known, but equally important, are the effects of selective attention when we use our muscles to move ourselves or other objects. To learn Mechanics is not to add rigour to common experience: it is to transform it.

Terms used in Mechanics have wider meanings in other contexts

Most of the concept names in Mechanics have very much wider and more diffuse meanings in everyday language, particularly in military, political and social contexts. They develop such rich connotations through repeated use in these contexts that they cannot be used without drawing with them associations which are alien to science. As people appear to have a quite universal tendency to think and talk in everyday language unless compelled by circumstances to be more precise, there is plenty of scope for these wider meanings to infiltrate speech and writing about Mechanics. Informal explanations and applications to real examples are particularly at risk.

Scientific terminology is not free from ambiguity.

Qualitatively different phenomena may be given the same name or may be inadequately differentiated. Some concepts do not have universally accepted definitions. The concept of force is particularly badly served in this respect. There does not appear to be a single definition of force which is used consistently across different phenomena, even by individual authors. In particular, *force* and *net force* (or *resultant force* or *unbalanced force*) are used interchangeably while scalar measures of state such as tension and pressure are described and treated as though they were vector forces.

Mechanics provides a description of the behaviour of idealized bodies. Application of these rules of behaviour to real bodies is often difficult.

Rather like teachers of Geometry, teachers of Mechanics try to do two things at once. One is to provide an account of the real world which is accurate and reliable enough to be used to solve practical problems. The other is to provide a self-consistent axiomatic system in which true statements are made about abstractions. Moving between the two requires skill, particularly as the same real body may be idealized in different ways. For example, an iron rod may be idealized (modelled) as a particle, a rigid body, a continuous elastic medium or as an assembly of particles. Applying one of these idealizations means ignoring, or seeming to deny, properties required by the others.

Some of these barriers to understanding are outside the control of teachers. Nothing can be done to prevent students from developing their own connotations for everyday terms. All that can be done is to accept that they are likely to exist and to continue to point out potential conflicts. Others could be eliminated by the adoption of a consistent set of definitions and interpretations. What follows is an analysis of salient problems and an attempt to provide an accessible set of such definitions and interpretations.

FORCE AND NET FORCE

In my experience, recent graduates and teachers of Physics have great difficulty in defining *force* and analyzing simple situations in a way which is consistent with their definition. In particular, most define it in terms of the rate of change of momentum of a body and then find it difficult to deal with bodies at equilibrium. This should not be surprising, for what they define is *net force*, and this is zero at equilibrium. As there does not appear to be any widely accepted definition for *force* itself, I offer the following:

> Two bodies *interact* if they affect each other in any way;
>
> In an interaction, each body experiences a *force*. The two forces are equal in magnitude but opposite in direction (not necessarily collinear);
>
> If two bodies interact in more than one way, each interaction is characterized by a pair of forces independently of any other interaction;
>
> The equality of the magnitudes and the opposite directions of the two forces in an interaction are required by the principle of conservation of energy.

This definition permits *net force* to be separately defined through Newton's second law. Net forces also exist in pairs which are equal in magnitude and opposite in direction, but this is required by the principle of conservation of momentum, and is expressed by Newton's third law.

EQUAL AND OPPOSITE FORCES: NEWTON'S THIRD LAW

Pairs of equal and opposite forces are found in at least five qualitatively different circumstances. This ought to mean that they are described and explained in five different ways in textbooks, but this is not the case. In particular, Newton's third law is applied indiscriminately to as many as four. The five circumstances, in their simplest forms, are:

1. a collision involving two bodies;
2. an interaction between two bodies at a distance and at equilibrium;
3. two bodies in contact and sharing the same acceleration;
4. two bodies in contact and at equilibrium;
5. a single body at equilibrium while experiencing forces arising from two separate interactions.

The fifth of these often causes difficulties to students who do not appreciate that the pair of forces in Newton's third law must act on different bodies but, as it is almost always correctly treated in textbooks, and as no-one supposes that the third law applies to it, it will not be considered further here. The only one to which Newton's third law applies unequivocally is the first. To apply it to any of the others is both to introduce an unnecessary argument and to contradict what Newton wrote.

1. Collisions

I am using "collision" in its general sense, to mean an interaction in which two bodies exchange momentum, whether this takes place over a short or a long time. In such a collision, each body is acted upon by a net force equal to the time rate of change of momentum of the body. Newton's third law states that these two net forces are equal and opposite. In Newton's own statement of the third law *action* and *reaction* represent impulses, not forces (McClelland 1993).

Of course, 300 years later we are not compelled to remain consistent with Newton, but any change should be consciously decided, and made for good reason. Being able to apply the law to qualitatively different circumstances is not a good reason. On the contrary, it adds to the likelihood of misapprehension and confusion.

2. Forces between bodies at equilibrium arising from a single interaction

Large bodies interact "at a distance" either gravitationally or electrically, and we say that each experiences a force. This interaction is independent of any other interaction which may affect the bodies and exists whether or not they are at equilibrium. If the forces were not equal, Newton argued that the interaction could be used to accelerate a third body indefinitely, which he saw a contrary to the first law, not the third. Newton's argument basically appealed to the principle of conservation of momentum. An argument based on the principle of conservation of energy is more general, for the forces must still be equal and opposite when there is no exchange of momentum. For central forces, the force experienced by one body as it interacts with the other is equal to the rate of change of potential energy of the system with separation of the bodies.

Given that Newton did not think that the equality of forces in an interaction was a consequence of the third law, and given that there is a perfectly good, and even more general, argument to show that they must be equal, it seems pointless and perverse to disagree with him. If a book rests on a table and I want to say that the gravitational force on the book due to the earth is equal and opposite to the gravitational force on the earth due to the book, I do not need to add "by Newton's third law". The statement could be taken as a defining characteristic of the interaction or it could be justified in terms of conservation of energy.

3. Bodies sharing the same acceleration

When a body is accelerated by a contact force, as when a person throws a stone or a car pushes a truck, a pattern of thrust stress is set up through the system. At a boundary, such as between the hand and the stone, or between the car and the truck, each body is distorted. It is often stated that each experiences a net force and that the pair of forces illustrate Newton's third law. The *Force Concept Inventory* (Hestenes 1992) includes two items (Nos. 11, 13) which depend on this idea.

In item 11 (Figure 1), when student "a" pushes with his feet, the feet share the acceleration of "b". The "particle" which accelerates to the left does not include the lower legs of "a", while the one which accelerates to the right includes more than "b". The lower legs of "a" experience a net force to the right. If they experienced a net force to the left they would accelerate to the left. As far as I can tell, absolutely nobody disagrees with the statements that:

> acceleration of a body requires a net force; and
>
> the net force and the acceleration are in the same direction;

so everyone should agree that it is quite absurd to suggest that the feet would accelerate in the opposite direction to a net force acting on them. Inertia is not a force, and "b" does not exert any net force on "a". The equal and opposite net forces are both applied by the muscles of student "a". The key to the item, a statement that "a" and "b" exert equal and opposite forces on each other, would suggest that the accelerations of the chairs should be inversely proportional to the masses quoted in the item. This is not so and, because the third law is a universal law, the situation cannot be retrieved by assuming that the legs and feet of "a" are of negligible mass.

Why the counter-intuitive misconception represented by item 11 should have arisen in the first place, and why it should have been perpetuated through generations of textbooks, is not at all clear. It may well stem from bad modelling based on uncritical acceptance of the proposition that, to accelerate a body, you need an "external force". The only thing "external" to "a" which could be thought responsible for the acceleration, is "b", so the force is ascribed to "b". But, as a space rocket demonstrates, a force of external origin is not necessary for acceleration of a body. All that is necessary is that the body should be able to re-define itself by discarding some of its mass. In item 11 the body consisting of "a" and "b", and initially at rest, can re-define itself by discarding "b" and, temporarily, a small part of "a". When contact is broken "a" undergoes a brief acceleration to the right as his lower legs gain momentum at the expense of the momentum of the rest of his body. The fact that the body which defines "a" as a person is not the same as the body which initially accelerates to the left is a further potential source of confusion.

Item 13 suffers from the same problem (Figure 2). The car cannot possibly experience a net force from the truck as its acceleration is in the same direction as the acceleration of the truck. The "particle" which accelerates in the opposite direction is partly visible in the figure, but is not mentioned in the text.

When real bodies are accelerated by contact forces a set of stresses must be set up to transmit the forces to every part of each body so that every part shares the same acceleration. The stress in the system, thrust stress in the two examples, can be described and calculated in its own right. When it is, the bodies are modelled as continuous and elastic, not as particles. Stresses should not be confused with the net forces acting on the particles to which the system can also be modelled. Any part of an accelerating body has a difference in thrust across it (or tension if it is being pulled) which is equal to the net force required to give it that acceleration, but the tension or thrust at a specific cross-section cannot be described as a pair of net forces. Models must be compatible but they must not be mixed.

4. Equilibrium

If it is accepted that Newton's third law applies only to net forces then, when bodies are in equilibrium and no net forces exist, there is nothing to which to apply the law, except in the trivial sense that zero equals zero. The first law is enough to assure us that, if a body is known to experience a force, it must experience others which balance it, for no acceleration means no net force. From the point of view of Newton's laws, a group of bodies which are in contact and at equilibrium constitute a single particle. If we single out some part of the group it is all too easy to forget about the rest, particularly when it consists of the Earth or something attached to it.

I am willing to offer a prize of US$100 to the first person to give me a situation for which this argument is not valid.

FORCE AND TENSION

A major over-simplification which is commonly found in textbooks is to treat scalar quantities, such as tension, thrust and pressure, as vector quantities. Equations mixing tensions and forces, and tensions "resolved into components" are so usual as to be standard. Tensions and thrusts are linear quantities but they are not directional. It is only differences in tension or thrust which have direction and so can be treated as forces. Very often a difference in tension across a body is between a real value and zero, so that the difference is equal in magnitude to the tension, but this should not lead to the real distinction between a scalar and a vector quantity being ignored or denied.

Unlike forces, which exist independently in pairs, tensions always depend on circumstances. A tension in a rope or spring under equilibrium conditions is not necessarily a good guide to the force available to accelerate part of the system when equilibrium is ended. For example, let a mass be attached to one end of a spring and let the other end of the spring be "fixed". Under free fall, if the spring is extended and the body held "at rest" the tension T in the spring will depend on its extension and its spring constant. If the body is released, the magnitude of the net force accelerating the body is always less than T and may be very much less, depending on the relative masses of the body and the spring.

One source of the problem may be that there is no adequate definition for tension. Such a definition must provide for a scalar quantity whose value can change along a body. I shall offer the following:

> When (an element of) a body is extended from a reference length and held at equilibrium, its tension is the rate of change of change of elastic potential energy of the (element of the) body with extension.

The more general concept, tensile stress, at a cross-section through a body then becomes the rate of change of energy density with strain.

FORCE AND PRESSURE

There are at least four conceptually distinct situations to which the term "pressure" is commonly applied:

1. Bulk stress in equilibrium, related to volume strain by the bulk modulus. This seems to be a universally accepted meaning.

2. The pattern of forces (or, more strictly, intensities of forces) needed to change the volume of a body without acceleration. This is not the same as 1.

3. Thrust stress. This is a common meaning for pressure in everyday language but there is no excuse for carrying it across into Physics. When a lady wears stiletto heels her weight, acting linearly, gives rise to thrust stress, not pressure, at the floor. Incidentally, contrary to most claims, this is seldom associated with visible damage: most occurs at the edge of the heel due to extension of the floor material.

4. The (areal) intensity of a net force. Pressure is often identified with molecular bombardment of the walls of a container of gas.

Multiple meanings constitute a recipe for confusion. Probably the greatest single villain is the common definition of pressure in terms of force/area. Pressure is a scalar quantity and cannot be defined through a single vector quantity. Even if *area* is taken to be a vector quantity, there is no meaning for the quotient of two vectors. Definitions in terms of gas bombardment are always unacceptable as they have no meaning except at a boundary. Within a gas, the average rate of transfer of momentum is zero. Pressure must also be definable within a condensed material, where, on average, molecules are in equilibrium. As negative pressures are common in the biological realm, the definition must permit them, but there is no meaning to be attached to a negative force.

For what it is worth, the definition which I am currently testing out is "pressure at a volume element of a material is the rate of change of elastic potential energy with volume strain". This lacks the pizzazz of the conventional definition but it may have the merit of being defensible.

A *pressure difference* is a vector quantity and constitutes the intensity of a force acting on an element of material of constant cross-section towards the lower pressure end. This statement is not equivalent to the statement that there is a different `force/area' at each end. A pressure difference can `act' and can be said to `cause' changes without introducing confusion. Pressure itself cannot be described in these ways. Textbook writers often associate pressure with verbs characteristic of force (McClelland 1987), but it is always a pressure difference which should be identified.

CONCLUSION

It may be that erroneous ideas about concepts in Mechanics are derived from naive experience and everyday language but, unless the information we present as teachers is internally consistent, it is unlikely to be a good competitor. As textbook writers persistently use terminology inconsistently and without drawing upon themselves reproof or correction, it must be assumed that teachers behave in the same way. The fact that some students transcend their instruction may need more explanation than the fact that many give up, and at least part of the explanation may lie in the fact that the mathematical problems which we routinely set for our students can be answered correctly without any serious conceptual grasp of the subject.

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