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How, How Often, and Under What Conditions

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Technical Editor's Note: Figures are located at the end of this article as supplied by the author.

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How, How Often, and Under What Conditions

Misconceptions are Developed: The Case of Linear Graphs

Abstract

The purpose of the present study was threefold: (a) to examine misconceptions related to the construction of linear graphs; (b) to analyze the cognitive processes underlying the development of the different kinds of misconceptions; and (c) to investigate the effects of a metacognitive intervention on students' sense of graphs. The intervention was derived from current theories in the areas of social cognition and metacognition, and from systematic observations of adolescents solving complex mathematical problems. Participants were seventh grade students randomly selected from two Israeli schools (four classes). Intact classes were randomly assigned into either an experimental or a control group. Results indicated that overall students encountered serious difficulties in constructing (and interpreting) linear graphs. Five kinds of misconceptions were identified: constructing an entire graph as a single, one point; misunderstanding the notion of covariation; conserving the form of an increasing function under all conditions; syntax errors; and confusing a graph and a picture. Further analyses indicated, however, significant differences between the experimental and control groups on students' sense of graphs. While the misconceptions were robust in resistance to "conventional" instruction, the metacognitive intervention facilitated the construction of a problem representation.

How, How Often, and Under What Conditions Misconceptions are Developed: The Case of Linear Graphs

Graphing has long been recognized as fundamental skills used to represent information. By its very nature, graphing involves both interpretation and construction. According to Leinhardt, Zaslavsky, and Stein (1990), interpretation refers to students' ability to read a graph (or a portion of a graph) and make sense or gain a meaning from it. Construction, on the other hand, refers to "the act of generating something new ... building a graph or plotting points from data (or from a function rule or a table). In its fullest sense, construction involves going from raw data (or abstract function) through the process of selection and labeling of axes, selection of scale, identification of unit, and plotting" (Leinhardt et al, 1990, p. 12). Leinhardt et al further explained that "construction is quite different from interpretation. While interpretation relies on and requires reaction to a given piece of data (e.g., a graph, an equation, or a data set), construction requires generating new parts that are not given" (p. 12).

Although graphing is an essential part of mathematics and science curriculum, consistent evidence has shown that students' sense of graphs is rather limited. Difficulties have been observed at all levels of education -- from elementary school to graduate studies, and at all levels of information processing -- from the lowest level involving data extraction (e.g., who gained the highest score in the class), through the intermediate level involving trends seen in parts of the graph (e.g., between the years 1990 and 1993, which company had the largest growth rate?), to the highest level involving an understanding of the deep structure of the data (Wainer, 1992). Yet, the current research suffers from two deficiencies. First, most of the studies in this area have been limited to graph interpretation; almost nothing is known at present on construction (Leinhardt et al, 1990). Second, the learning environments used to enhance students' sense of graphs have not been adequately grounded in theory and research. The purpose of the present study was to provide a mapping of students' misconceptions in the area of graphing, and to investigate the effects of a metacognitive intervention on students' sense of graphs.

Nesher (1987) defined a misconception as “a line of thinking that causes a series of errors all resulting from an incorrect underlying premise rather than sporadic unconnected and nonsystematic errors” (p. 35). Thus, misconceptions are not sloppy mistakes or slips, but rather systematic errors that reappear whenever the same type of problem is presented.

Nesher’s (1987) definition raises the question of how misconceptions are developed. Several researchers hypothesized that misconceptions result from a simple lack of knowledge that if provided would extract the error. Others assumed that misconceptions result from overgeneralization of preexisting knowledge. Research has shown, however, that both hypotheses should be rejected. Students who were exposed to the appropriate knowledge were not able to recognize that the procedures they apply are erroneous nor could they understand why a procedure that was correct in one domain is incorrect in another domain. VanLehn (1983) explained that when solvers encounter a new problem, they try to apply their preexisting knowledge to the new situation. If they fail to solve the problem, they may introduce a “repair” in the procedure. When the changed procedure is correct, a creative solution is obtained. However, when the changed procedure is incorrect -- a misconception is manifest. Thus, according to the “repair theory”, misconception is not a simple overgeneralization that occurs whenever the solver applies a concept as is to other domains in which it is incorrect; a misconception results from a slight repair to a given procedure. As Resnick et al (1989) pointed: “[misconceptions] are intelligent constructions based on what is more often incomplete than incorrect knowledge” (p. 26).

What are the misconceptions in the area of graphing? How are they developed? And under what conditions can they be extracted? The present research was designed to address these questions. The research consists of two studies: a large scale study that focused on the mapping of students’ misconceptions, and an empirical study that examined the conceptual changes of students exposed to two different learning environments.

Study I

The major purposes of Study I were: (a) to examine misconceptions related to the construction of linear graph; and (b) to analyze the cognitive processes underlying the development of the different kinds of misconceptions. Mapping students' misconceptions in the area of graphing was currently conducted by Mevarech and Kramarsky (under review). In this study, seventh and eighth grade students were asked to transform verbal descriptions into graphic representations. Given this study, we wanted to examine the extent to which misconceptions previously found in the area of graph interpretation would be observed also in the area of construction. In addition, we wanted to further investigate the misconceptions related to construction by conducting a large scale study, based on a sample drawn from various schools and representing students of different ability levels.

Method

Participants

Participants were approximately 400 students randomly selected from five Israeli junior high schools. Since the major purpose of this study was to provide a mapping of students' misconceptions in the area of linear graphing, no attempts were made to compare the effectiveness of various learning environments on students' sense of graphing.

Instrument and Procedure

The instrument was the same as that used in the study of Mevarech and Kramarsky (under review). Subjects were presented four situations: the first referred to an increasing function, the second to a constant function, the third to a curvilinear function, and the fourth to a decreasing function. Subjects were asked to plot on blank paper, graphs representing each situation. Below is the description of the four situations:

> "Sarah, Rivka, Rachel, and Leah discussed the question of whether or not their success on tests is related to the amount of time that they prepare for the tests. Sarah claimed that the more she studies, the better her grades are. Rivka argued that no matter how long she

studies, she always gets the grade 10. Rachel, however, said that when she studies up to three hours, the longer she studies the better her grades; but, beyond three hours, she becomes tired and her grades are lower. Leah confessed that for her, generally, when she studies more, her grades decrease."

As in the study of Mevarech and Kramarsky (under review) also in this study, all students (100%) transformed the verbal descriptions directly into a graph without calculating any specific points. Furthermore, although the situations did not necessarily refer to linear functions, all students (100%) preferred to describe them as such. Students were not allowed to use rulers and/or computer programs to plot the graphs.

Results and Discussion

As in our previous study (Mevarech and Kramarsky, under review), also in the present study, junior high school students encountered considerable difficulties when asked to transform verbal information into graphing representation. Generally speaking, the misconceptions identified in the previous study, were replicated in the present study. To get a more refine mapping, however, we categorized the misconceptions into five categories, described below: (a) constructing an entire graph as one, single point; (b) misunderstanding the notion of covariation; (c) conserving the form of an increasing function under all conditions; (d) syntax errors; and (e) iconic graphing.

Constructing an Entire Graph as One Point: The "one point misconception" was observed when some students correctly plotted the axis system, but instead of drawing a line or a series of bars, they marked only one point (Fig. 1) or one histogram (Fig. 2). A modified version of the "one point misconception" was observed when students plotted the four situations in one graph, each situation was represented by a single histogram (Fig. 3). Another modified version was manifest when students represented the constant function by one point connected to several ticks on the x axis (Fig. 4).

The "one point misconception" is the most primitive model. It reflects students' notion that a changing situation can be represented by one, single point. This misconception may result from students' initial exposure to ordered-points when they learned about graphing. It may also result from students' everyday knowledge, that in a changing situation, the most important event is the last event. Indeed students possessing the "one point misconception" tended to represent the increasing function by a point constructed of high values on both variables, and the decreasing function by a point constructed of a high value on one variable and a low value on the other variable. Interestingly, analyzing students' works showed how the "repair" was induced. The students first plotted a "small" histogram. On the top of that histogram, they plotted a larger histogram and once again a larger histogram.

Insert Figures 1-4 About Here

Misunderstanding the Notion of Covariation: The "covariation misconception" was observed when students could not relate to several sources of information simultaneously. For example, some students conceived a graph as representing just one variable; in drawing graphs, they used only one "axis" and matched the values marked on both sides of the "axis". Others, represented a changing situation by using a series of axis systems, each represented only one single point. Still others separated the curvilinear function into two graphs: one represented the increasing interval, and the other, the decreasing interval.

The "covariation misconception" may be considered as an elaboration of the very primitive model of the "one point misconception". Students who possessed the "one point misconception" but realized that a changing situation cannot be represented by a single point (or a single interval), induced a "repair" in their basic procedure: they represented the situation by a couple of points, each plotted on a different axis system.

Conserving the Form of an Increasing Function Under All Conditions: The "increasing function" misconception was observed when some students conceived all linear graphs as having the form of an increasing function (i.e., $y=ax+b; a>0$).

To keep the graph in the "proper" upward position, students induced all kinds of "repairs", most related to changing the scales on the axes. For example, the constant function was represented as an increasing line by marking only the value of the constant on one of the axes (Fig. 5). The curvilinear function was represented as an increasing line by using nonsequential values on one of the axes (Fig. 6). The decreasing function was represented as an increasing function either by using a mirror image of a graph (e.g., the *y* axis was on the right side of the *x* axis), or by reversing the order of the values on one of the axes (Fig. 7). Also in this case, as in the case of the "one point misconception", analyzing students' works showed how the repairs were induced. For example, many students initially marked the scale correctly, but then remarked it incorrectly in order to be consistent with their misconception.

Insert Figures 5-7 About Here

Syntax Errors: Graph is a symbol system with its own language, grammar, and syntax. We already described syntax errors associated with the "increasing line misconception". Another kind of syntax error, frequently found in students' construction relates to connecting the ticks on the *x* and *y* axes by using either straight ("diagonal") lines or curved lines, rather than the distance from each of the axis (Fig. 8).

Insert Figure 8 About Here

Iconic Presentation: The naive concept of confusing a graph and a picture was illustrated when some students plotted a graph and framed it, although the graph did not display the information described in the narrative. Other students plotted a graph as an arrow: a vertical arrow pointing upward represented an increasing function, an arrow pointing downward represented a decreasing function, and an arrow pointing to the right represented a constant function. Other iconic representations included: steps, names, etc.

The findings of this study extend the results of previous studies showing the difficulties students encounter in transforming verbal descriptions into graphic representation. Although students in junior high school are exposed to graphs not only in classrooms, but also in newspapers, advertisements, and other media, their sense of graphs is based on primitive models and naive conceptualizations. The question that remained open relates to the design of a learning environment that would facilitate the understanding of graphs. Study II addresses this issue.

Study II

The importance of graph understanding led us to investigate the question of the extent to which such misconceptions could be extracted by using an intervention based on metacognitive questioning applied in small heterogeneous groups and guided by feedback-corrective procedures. The teaching method is called IMPROVE. It is the acronym for:

Input: Introducing new concepts, skills, or procedures;
Metacognitive Questioning;
Practicing and testing;
Reviewing;
Obtaining mastery on higher and lower order thinking skills;
Verifying the attainment of mastery;
Enrichment activities.

The IMPROVE method was initially designed by us to improve mathematical thinking, mathematical language, and self monitoring of problem solving. It is based on current theories in the areas of social cognition and metacognition, and from systematic observations of adolescents solving complex mathematical problems. The method is currently implemented in about 150 heterogeneous classes in Israel. Empirical evidence has shown that students who used the IMPROVE method scored significantly higher on mathematics achievement than their counterparts who did not use the method. Compared to the grouping method, the positive effects of IMPROVE were observed for high as well as low achieving students.

While there has been promising evidence regarding the effects of IMPROVE on mathematical achievement, almost nothing is known at present on the effects ofthis method on students' misconceptions. Research indicates that learning environments appropriate for changing students' conceptualization should possess the following characteristics: (a) facilitation of both problem solving and metacognitive processes; (b) emphasis on social-cognitive interactions based on conflict resolution; and (c) provision of systematic feedback-correctives that point out the erroneous procedures (Confrey, 1990; Mevarech & Light, 1992). Given this research, we hypothesized that IMPROVE would significantly enhance both interpretation and construction of graphs (consonant with the metacognitive theories), and that effects would be particularly stronger on those aspects of graphing which relate to the social-cognitive interactions in the classroom (e.g., the use of mathematical language in explaining one's reasoning).

Method

Participants

Participants were 108 seventh grade students who learned in four Israeli junior high schools. The four schools were randomly selected from a list of 15 schools either learning with the IMPROVE method or serving as a control group. Intact classes were randomly assigned into one of two treatment groups: an experimental group (two classes; N= 57) and a control group (two classes; N=51).

Measures and Procedure

Three assessment measures were used in the present study: (a) Graph Interpretation Test; (b) Graph Construction Test; and (c) Mathematics Achievement Examination.

Graph Interpretation Test: The test included 22 items, each representing a graph describing linear functions. Students had to interpret the graphs and explain their reasoning. The test was designed on the basis of our observations regarding the misconceptions acquired by a sample of 400 students drawn from the same population (see Study I). The test was administered to all students prior to, and at the end of the study.

Graph Construction Test: This test was the same as that used in Study I. It included four items, each describing an everyday situation. Students were asked to draw the graphs representing the situations. The graphs represent an increasing function, a constant, a curvilinear function, and a decreasing function. This test was administered at the beginning and the end of the study. In addition, at the end of the study, students were asked to transform information represented numerically or formally into graphic representations. Thus, the scores on the pretest ranged from 0 to 4, and the scores on the posttest ranged from 0 to 6.

Mathematics Achievement Examination: Prior to the beginning of the study, all students were administered a mathematics achievement test related to the ongoing mathematics curriculum. The test included 38 multiple choice and "open-ended" items. Analysis of students' performance indicated no significant differences between and within treatment groups (F values <1.00; $p>.05$). (The mean values of the two classes constituting the experimental group were: 66.4 and 67.6; standard deviations were: 19.9 and 16.5 respectively; the mean values of the two classes constituting the control group were: 66.25 and 67.0; standard deviations were: 18.3 and 18.6 respectively. The scores were calculated as percent of correct answers.)

Treatment

At the beginning of the study, all students were administered the two tests assessing the two facets of graph comprehension: interpretation and construction. Then, classes started to study the graphing unit according to the treatment into which they were assigned. The two treatment groups learned the same concepts, skills and procedures, for the same duration of time. They differed, however, only in the teaching method.

The IMPROVE classes learned in small heterogeneous groups of four students: one high, one low, and two middle achievers. While solving the problems, students were trained to use the following metacognitive questioning: What is the problem all about? What are the quantitative and qualitative strategies appropriate for solving the problem? and What are the differences between this problem and the previous problem you had solved? Students were exposed to feedback-corrective-enrichment as part of the IMPROVE method.

The control group learned the unit by using the conventional instruction based on student-teacher dialogs.

Results and Discussion

The analyses were conducted in two steps. First, the two groups were compared on pretreatment measures. Then, a one-way Analysis of Covariance (ANCOVA) was performed with pretreatment scores used as a covariate.

Constructing Linear Graphs

Table 1 presents the mean scores and standard deviations of the construction tasks by treatment and time. As one may see from Table 1, prior to the beginning of the study, the two treatment groups performed quite the same. The overall mean scores of the experimental and control groups were 2.0 and 2.08; (SD=1.56 and 1.43) respectively; $F(1, 98) < 1.00$.

Insert Table 1 About Here

Although no significant differences were found between groups prior to the beginning of the study, large group differences were observed at the end of the study. As indicated in Table 1, the overall mean of the experimental group was 3.85 (SD=1.93), while that of the control group was only 3.19 (SD=1.59). These differences were statistically significant ($F(1, 98)=4.07$; $p<.05$).

Analyzing students' performance on each task separately indicated the beneficial effects of IMPROVE on students' ability to construct an increasing function and a curvilinear function. In addition, IMPROVE students were better able than the control group to transform information from algebraic representation into graphic representation. These differences were statistically significant.

Graph Interpretation Tasks

Table 2 presents the mean scores and standard deviations on the graph interpretation test by time and treatment. As one may see from Table 2, students' ability to interpret linear graphs prior to the beginning of the study was rather low. In addition, Table 2 shows that the two treatment groups performed quite the same on the pretreatment test The mean scores of the experimental and control groups on the pretreatment test were: 11.43 (SD=4.22) and 10.44 (SD=4.31) respectively; scores ranged from 0 to 22. These differences were statistically not significant .

Insert Table 2 About Here

Table 2 further indicates that the end of the study, students exposed to IMPROVE tended to score higher on the interpretation test than their counterparts in the control group. The mean scores were 13.47 (SD=3.54) and 11.72 (SD=3.63 for the experimental and control group respectively. These differences were statistically significant (F (1, 105)=4.86. p<.03).

Detailed analysis of students' misconceptions indicated that thein the control group, the percent of students who hold misconception was larger than that in the experimental group. Chi square analyses showed significant and marginal significant differences on misconceptions related to the curvilinear function and the constant function respectively.

Further analysis focused on students' use of mathematics language in explaining their reasoning. This analysis showed that although no significant differences were found between the two groups prior to the beginning of the study (mean scores= 2.11 (SD=2.31) and 2.75 (SD=2.74 for the experimental and control groups respectively; F<1.00) large differences were found at the end of the study even when pretreatment differences were controlled (F(105)=7.85, p<.006). ANCOVA showed that IMPROVE students performed higher on this variable than the control group (Mean scores= 5.41 (SD=3.28) and 3.63 (SD=2.38) for the experimental and control groups respectively.

General Discussion and Conclusions

The present research incorporates in a large number of studies showing that students did not enter the learning situation as a tabula rasa (e.g., Confrey, 1990). Many students possessed misconceptions that exert detrimental effects on further learning. Analyzing the misconceptions related to graph construction revealed five kinds of misconceptions: (a) constructing an entire graph as a single, one point; (b) misunderstanding the notion of covariation; (c) conserving the form of an increasing line under all conditions; (d) syntax errors; and (d) confusing a graph and a picture. These misconceptions were rather robust in resistance to conventional instruction on graphing.

The findings of the present research provide a broader perspective to analyzing misconceptions related to graphing. Both studies showed that many students' in seventh grade possessed a very primitive model of graphing. That model refers to conceiving any graph as a one single point. Students who realized that a changing situation cannot be presented by one single point, induced a “repair” in that primitive model: they presented the situation by two graphs, each represented one, single point. Further “elaboration” of the “one point misconception” was observed when some students split the curvilinear graph into two graphs: one representing an increasing function and the other a decreasing function. This primitive model may also explain the misconceptions previously suggested by Leinhardt et al (1990). In their excellent review, Leinhardt et al (1990) described how students at various levels of education confuse a slope and a height, an interval and a point, and a graph and a picture. In addition, Leinhardt et al pointed out that many students cannot consider graphs as a continuous entity; they tend to look on graphs pointwise. It may be that these misconceptions are also derived from the primitive model of “one point misconception”.

The present study provides further support to VanLehn “repair” theory. Qualitative analyses of students’ misconceptions showed how students applied their preexisting knowledge to drawing graphs. When that knowledge was inappropriate, students induced a “repair” in the procedure. Often, the “repaired procedure” was inaccurate. That did not prevent students from applying the incorrect procedure (i.e., misconceptions) whenever they had to construct or interpret graphs. The way students induced the repairs could be observed in students’ works.

The interference of everyday knowledge in constructing and interpreting graphs was frequently manifest in the present research. For example, students tendency to connect the ticks on the x and y axes by using “diagonal lines” may result from their natural approach to connect two points by straight lines. Also students’ tendency to interpret and construct graphs as pictures, may result from their everyday approach to icons.

Further analyses showed the beneficial effects of the IMPROVE method on interpreting and constructing graphs. The IMPROVE method is based on a metacognitive intervention employed in small heterogeneous groups and guided by a feedback-corrective procedure. Although some of the misconceptions described above did not change under formal instruction, the number of students who were able to overcome the difficulties and attain mastery on graphing was much larger under the IMPROVE condition, than under conventional instruction. These findings incorporate in a series of studies showing the positive effects of a metacognitive intervention on students' problem solving skills and creativity (Mevarech & Kramasrky, 1992). The findings also extend the results of previous studies showing the positive effects of cooperative learning embedded within feedback-corrective procedures on mathematical thinking (Mevarech, 1985, 1991) and higher cognitive processes (Mevarech & Susak, 1993). Future research may focus on other learning environments and may find new ways to facilitate the understanding of graphing. This topic merits future research.

Table 1: Mean scores and standard deviations on the construction text by time and treatment.

	IMPROVE GROUP	CONTROL GROUP
Pretest M:	2.0	2.08
SD:	1.56	1.43
Posttest M:	3.85	3.19
SD:	1.93	1.59

Table 2: Mean scores and Standard deviations on the interpretation test by time and treatment

		IMPROVE GROUP	CONTROL GROUP
Pretest	M:	11.43	10.44
	SD:	4.22	4.33
Posttest	M:	13.47	11.72
	SD:	3.54	3.63

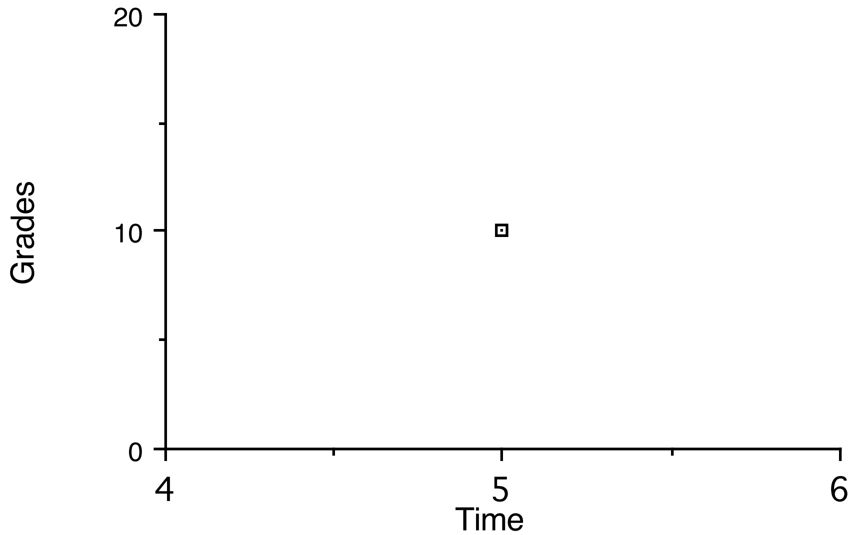

Figure 1: Misconception related to the representation of an increasing function: Representing an entire graph as a one, single point.

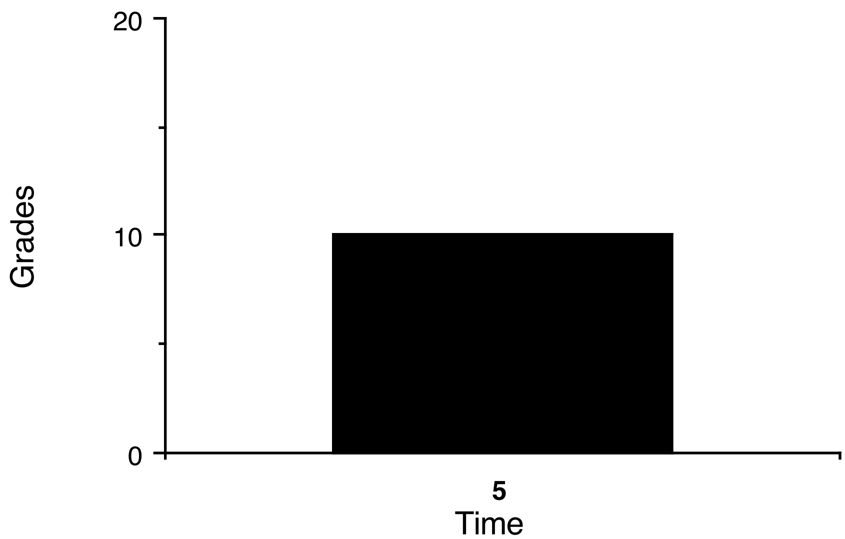

Figure 2: Misconception related to the representation of an increasing function: Representing an entire graph as one, single histogram.

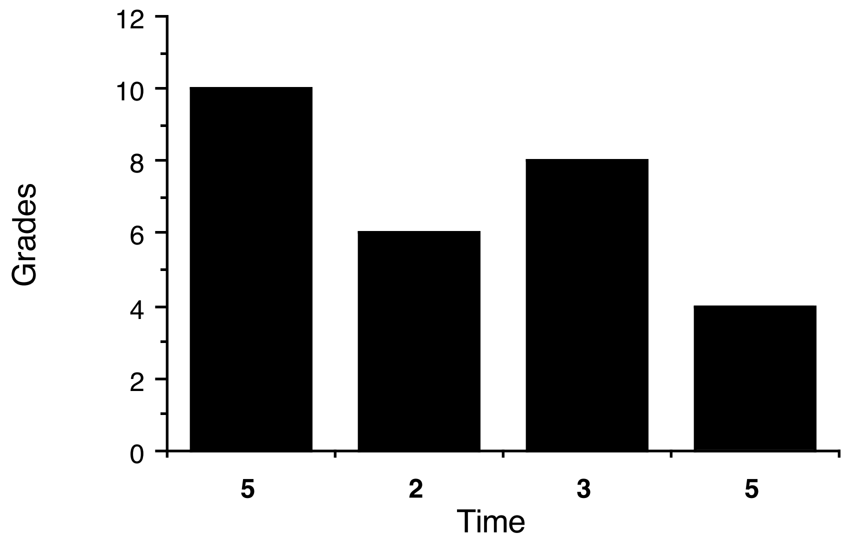

Figure 3: Representing four changing situations by four histograms: A modified version of the "one point misconception"

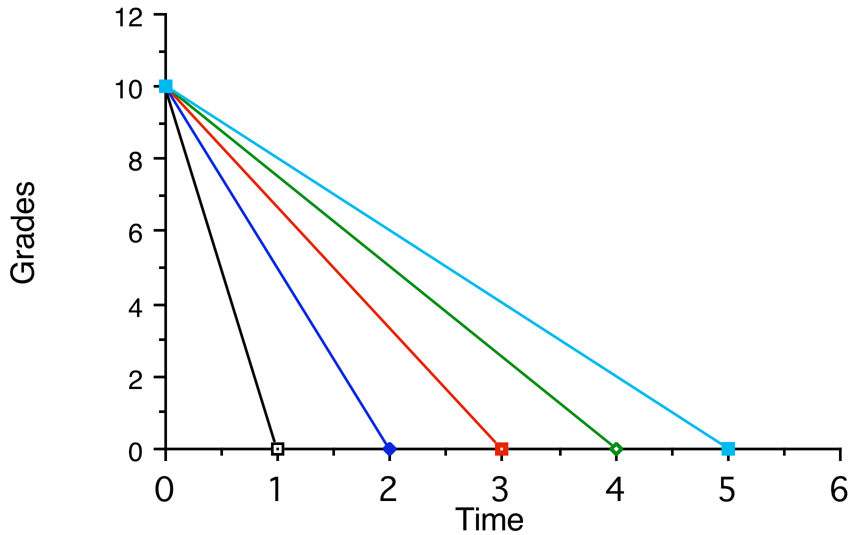

Figure 4: Misconception related to the representation of a constant function: A modified version of the “one point misconception”

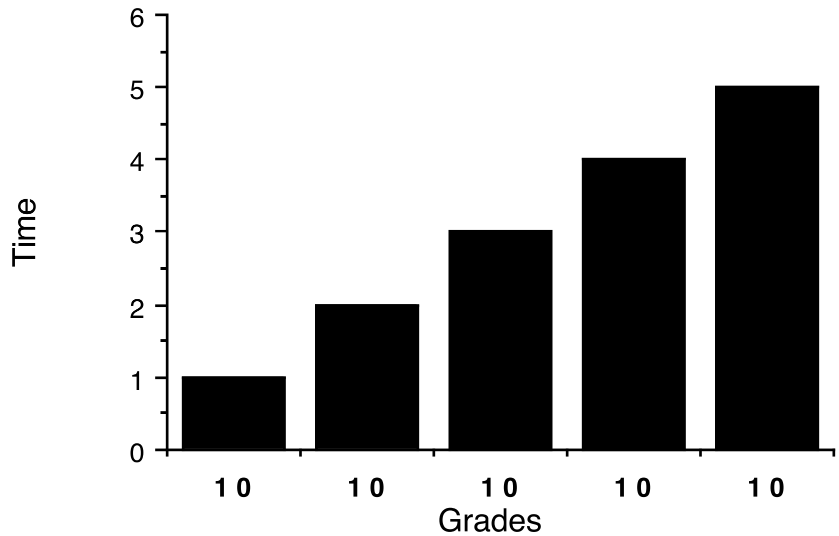

Figure 5: Representing a constant function as an increasing function by using only one values on the x axis.

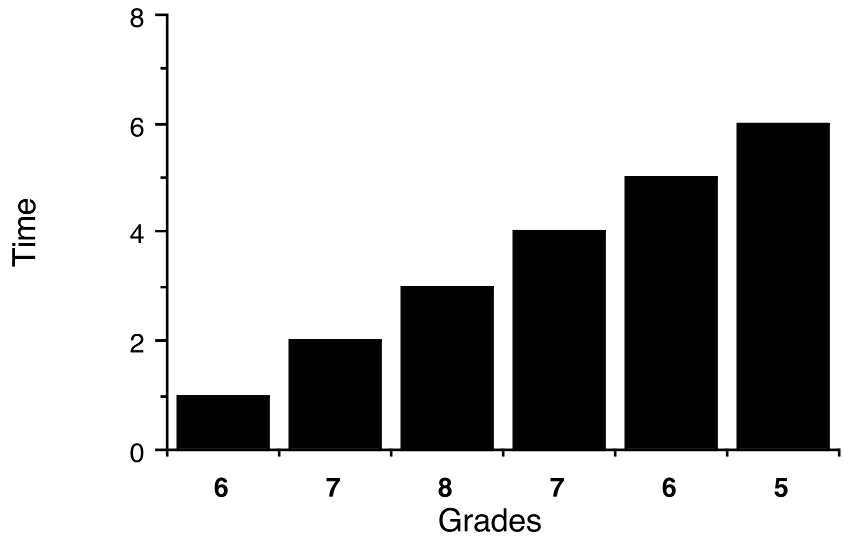

Figure 6: Representing a curvilinear function as an increasing function by using nonsequential values on the x axis.

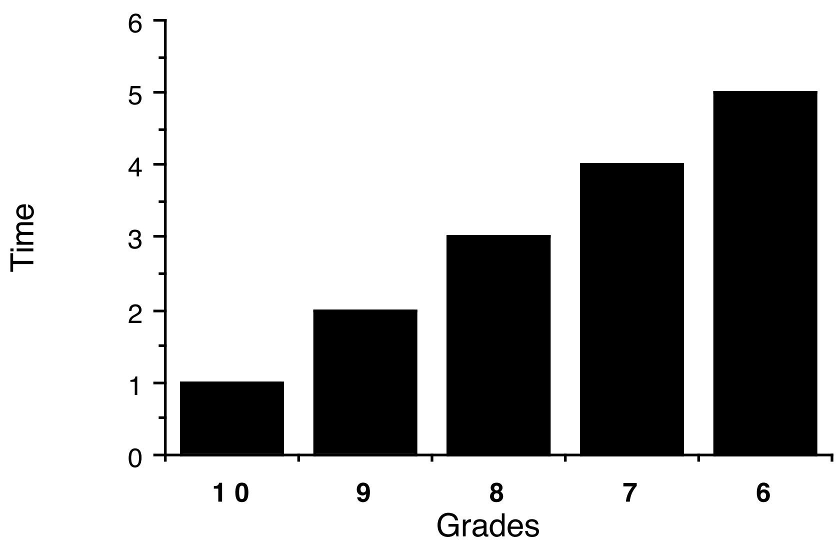

Figure 7: Representing a decreasing function as an increasing function by reversing the values on the x axis.

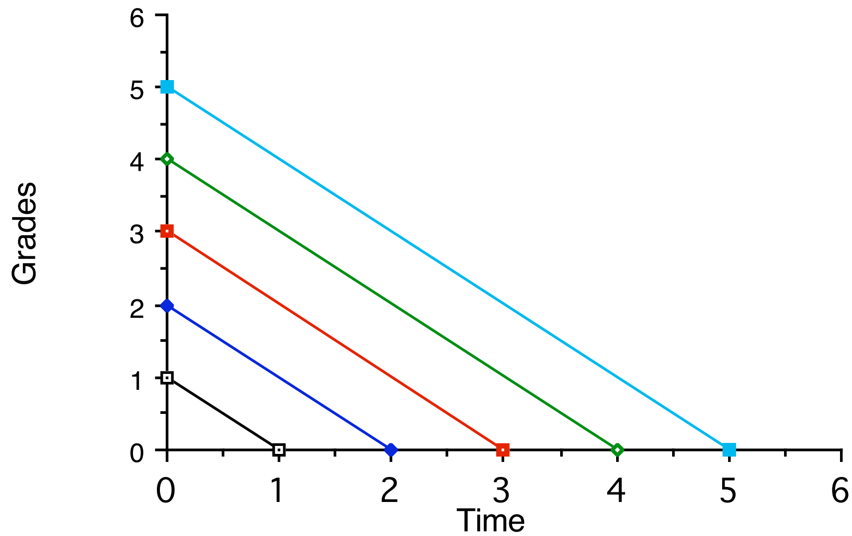

Figure 8: Syntax error: Representing an increasing function by straight, diagonal lines connecting the points on the axes.

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