Third Misconceptions Seminar Proceedings (1993)

Paper Title: **MISCONCEPTIONS IN MATHEMATICS AT UNDERGRADUATE LEVEL**

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Keywords: Concept Formation,,,Misconceptions,Mathematical Concepts,Concept Formation,Comprehension,Cognitive Psychology,Cognitive Ability
General School Subject: Mathematics
Specific School Subject: Algebra
Students: Undergraduate

Macintosh File Name: Ruberu - Mathematics
Release Date: 1-11-1994 D, 11-7-1994 I

Publisher: Misconceptions Trust
Publisher Location: Ithaca, NY
Volume Name: The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics
Publication Year: 1993
Conference Date: August 1-4, 1993
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

A Correct Reference Format: Author, Paper Title in The Proceedings of the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Misconceptions Trust: Ithaca, NY (1993).

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MISCONCEPTIONS IN MATHEMATICS AT UNDERGRADUATE LEVEL

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INTRODUCTION

The usefulness of schematic learning in mathematics (Skemp, 1971) at all levels, cannot be over-emphasized. Nowhere is this more relevant than at university, at which level mathematics assumes a particularly high degree of abstractness. The abstract conceptual nature of mathematics has been pointed out by many writers (Skemp, ibid.; Collis, 1972) as the chief reason why mathematics learning is more demanding at all levels.

Unfortunately, however, many students even at undergraduate level, seem not to appreciate this vital need for learning mathematics with understanding. Instead, they would settle for quick, ready-made answer getting techniques such as the use of short-cuts, stock formulae and other rote procedures, and an indiscriminate use of calculators. Making the grade at examinations is the prime consideration, no matter how! A survey conducted by the University of Brunei Darussalam on its own students in 1991, puts this succinctly thus:

> "...The motives for being in their present programmes are for the purpose of obtaining higher qualifications for gainful employment and to become specialised. The implication for this is that learning that is extrinsically motivated leads a student to "cut corners" with attention focused only on passing the hurdle and getting the reward with little effort involved..." (UBD Survey Subcommittee, 1991, p.i).

Consequently, there is widespread misunderstanding and a poor grasp of essential mathematical concepts and principles. Few would realise that this easy-going approach is not conducive to learning mathematics with understanding.

This paper is aimed at highlighting some of the more commonly held misconceptions and the use of inappropriate procedures in mathematics prevalent among undergraduates, and the probable causes thereof.

THE SAMPLE

The sample of undergraduates observed in this study comprised (a) 50 students who had reached the end of their three year training period on the Certificate in Education (Primary) Programme and (b) 14 students in the BA (Primary) Upgraders Programme in the Faculty of Education at the University of Brunei Darussalam.

The Certificate in Education is a three year non-degree programme mainly for pre-service training of young people as Primary school teachers. The minimum entry qualification to the programme is GCE Ordinary Level. The BA (Primary) Upgraders Programme is a two year degree programme meant for experienced Primary teachers.

METHODS OF COLLECTING DATA

The two main techniques of collecting data in this study have been the Diagnostic Interview (Skemp, 1981; Ruberu, 1986; 1992) and the Teaching Experiment (Firth, 1981).

An analysis of students' written responses to regular tests and assignments has also been conducted. This had as its aim, the probing of likely causes of errors and inaccuracies committed by them.

Similar observations made over the years, by other members of staff of the Science and Mathematics Education Department at UBD, have also been used.

THE RESULTS AND FINDINGS

Errors resulting from misconceptions and other deficiencies in many areas of mathematics have been observed. The use of inappropriate algorithms, misunderstanding of essential concepts and principles, blind adherence to rote bound procedures and a reluctance to adequately describe even correct procedures have been noted. Inadequate training in step-writing and presentation of ideas and an apparent disregard for the precise meaning of even elementary mathematical symbols were also noticeable. The writer has observed similar deficiencies in students at several institutions in Australia and Papua New Guinea, as well (Ruberu, 1987).

These inadequacies and probable causes thereof, are described under a few subject headings for convenience of reporting.

ARITHMETICAL CONCEPTS AND SKILLS

Ability to handle arithmetical computations successfully was deficient in many students. As Allendoerfer (1965) had once remarked, these students knew how to multiply and divide but not when to multiply and when to divide. Obviously, this is a malady resulting from a lack of understanding of the concepts involved.

The concept of Percentage (together with the associated skills) is a case in point. The direct computation of the percentage of a quantity was relatively easy. But the inverse operation of obtaining the quantity when some percentage (p %) of it was given, caused problems. "Do I multiply by p and divide by 100, or is it the other way round?".

Similarly, to obtain a number when some fraction of it was given, caused problems to many. Here, the concept of proportion (together with the associated skills) has not been acquired properly. A good deal of research has been done on the concept of Proportionality (Karplus, Karplus, Formisano and Paulsen 1975; Lunzer and Pumphrey, 1966). The findings of these studies seem to apply even to the undergraduate level.

Simplification of expressions involving decimals and/or fractions was a weak area too. The lack of an adequate repertoire of number facts seem to retard the progress of many students. A heavy dependence on the use of the calculator appeared to contribute to this. Many would start pressing the buttons even before reading a question properly! Some needed a calculator even to divide by 20.

Some students appeared to be concerned when a division resultsumber divided. That there are four "thirds" in one and one third has not been properly conceptualised (Inder, 1982, p.40). Language deficiency due to the lack of an adequate verbalisation of certain mathematical forms appears to be a likely cause of this. A lack of proper weaning from the use of certain empirical props in the development of these concepts early on, is also indicated here.

Some students appeared not to have progressed beyond the Piagetian "Concrete Generalisation" stage (Collis, 1971). Performing two operations simultaneously was a great burden for them.

ALGEBRAIC CONCEPTS AND SKILLS

Certain naively held algebraic misconceptions were often the cause of many errors of simplification. Some of the more frequently held ones are:

$(a+b)^2 = a^2 + b^2$, $a^3 + b^3 + c^3 = (a+b+c)^3$, $p/q + q/p = (p+q)/pq$

$\overline{a+b} = \overline{a} + \overline{b}$

Another frequently occurring weakness was the inability to apply a known principle to a situation appearing in a slightly different form.

One would be familiar with the well known theorem on the Difference of Two Squares, but is not able to recognise it when the x and y are replaced by two pure numbers. Also, there were those who could "see" a relation in one direction, but not in the opposite direction! Consequently, many a student had to labour hard to work a product such as $(-b+ \quad b^2 -4ac)/2a$ x $(-b- \quad b^2 -4ac)/2a$.

An equation of the form $x^4 - 5x^2 + 4 = 0$ "cannot be done" because it is a fourth degree equation! That it could first be reduced to a familiar form is recognised only by a few.

To solve the equation $(3x + 1)^2 = 4(x - 1)^2$, one must begin with opening the brackets. An alternative procedure which could yield the solution with much less work, is seen only by a few. The slavish adherence to "stock procedures" and certain canonical forms was evident here. (See also Firth, 1978).

The skill of factorising polynomial expressions - a fundamental prerequisite in most simplifications - was lacking in many.

If the digits of a number are x and y, the number is "xy"! Only when x and y were replaced by numbers, together with the reminder that "xy means x times y", the howler was recognised. That the number should be 10x+y or 10y+x was seen only by a few.

There were quite a few students who, even after being introduced to the above models, could not write down the correct expression for a three digit number to the base ten! The inability or the lack of "readiness" to use letters as generalised numbers, was also evident here.

The concept of a variable and the ability of representing a variable by a letter were not well developed in many students. The high degree of understanding required to interpret a letter as a variable is, of course, a slow process which is highly dependent on good teaching at the Secondary school level (Hart, 1981, p.105).

Permutations and Combinations were a particularly hard subject to understand, even when confined to simple exercises. So was probability. Despite repeated instruction from school days, there were many who did not have a clear grasp of the meaning of that term. There were those who would state in reply to a question that the probability of an event is 73.5, without any compunction!

GEOMETRICAL AND SPATIAL CONCEPTS

Several serious deficiencies were detected in these areas. Many students did not seem to have the ability to draw or visualise the correct figure as demanded by a question. Language deficiency seemed to be a heavy contributory factor here. The translation of verbal information into mathematical form was a difficult task for many a student.

Elementary geometrical concepts and principles associated with simple figures were not clearly understood by some students.

A perpendicular could only be visualised in the context of vertical and horizontal lines. Many failed to recognise the geometrical properties of a figure when the figure was presented in a different orientation. A classic example of this was the inability to correctly identify trigonometric ratios when the "opposite side" of the right triangle was not vertical. Not all the altitudes of an obtuse-angled triangle were noticed by some students. Learning disabilities such as the above, caused by "orientation factors" have been identified elsewhere too but at a lower level of pupil maturity (Hershkowitz and Vinner, 1984).

"Mathematical Proof" was often confused with conjecture or particular cases of illustration. Empirical evidence in support of a proposition was often held out as theoretical proof. The nature of proof seemed to be a particularly hard subject to comprehend. That a "proof" should cover every possible case under its domain, was recognised only by a few.

CONTRIBUTORY FACTORS

Several factors could be identified which seemed to contribute to the afore-said maladies.

(1). Lack of a clear grasp of essential concepts and principles.

(2). Lack of an adequate repertoire of number facts and associated skills.

(3). Inability to generalise concepts and principles beyond known boundaries.

(4). A strong desire for answer-getting or to somehow "get there" no matter what method was used.

(5). A heavy dependence on rote procedures and a slavish adherence to stock recipes, formulae and "short-cuts".

(6). Poor presentation of material : not stating necessary steps or the reasons for a step.

(7). Poor writing styles which often led one into error in computations. For example: writing $x = -b \; \underline{+} \; \underline{b^2 - 4ac}$ (for the formula for solving quadratic equations).

$2a$

(8). Incorrect or inappropriate use of mathematical symbols. For example, to obtain x from the relation $x^2 = 9$, one would write $x^2 = 9 = 3$. Here no thought was given to the fact that the same x^2 cannot also be equal to 3, or to the howler "9 = 3".

(9). Inability or unwillingness to appreciate the use of a novel method or a different approach for tackling a problem.

(10). Lack of mathematical curiosity or interest in "Open-Search" or creative thinking.

(11). Lack of an adequate level of language facility, especially for translating verbally presented information into correct mathematical forms.

(12). Over dependence on empirical props, and distracting influence by certain canonical forms.

PROBABLE CAUSES

The factors listed above could probably be regarded as precipitating causes. They seem to stem from much deep-seated deficiencies in these pupils, which could well be regarded as predisposing causes.

(1). Incomplete or inadequate acquisition of essential basic mathematical concepts and principles.

Many a concept or principle seemed to have been abstracted only partially. A bare association and no generalisation (or proper fixation) seemed to be there. Also there was no proper weaning from the use of concrete materials in procedures adopted at the initial stages of acquiring basic mathematical concepts.

(2). Acquisition of many basic mathematical skills and facts through rote-bound procedures.

This is related to (1) above. When concepts and principles are not abstracted through proper procedures, rote learning takes place (Skemp, 1964). Mathematical skills and facts derived through such learning will be limited to the contexts through which they were learnt, with little transfer of training being possible. A more serious consequence is that rote learnt material is not conducive to further learning of mathematics (Skemp, 1971). Also it encourages the use of rote-bound procedures even further!

Sometimes the use of rote-bound procedures seem to pass on from the teacher to the children in the classroom. A classic example of this was noticed by the writer while observing a UBD student's practicel teaching. This B Sc (Education) student was computing the "force constant" k of an elastic string with his class, using the equation $F = kx$. The force F was in newtons and the extension x was in metres. Having calculated the value of k, he wrote it down with units $Kg.s^{-2}$, and the children were obviously puzzled with these units. The writer called him to a side and discreetly told him that the units could be newtons per metre as well, and that they were more meaningful in this situation. Now it was the teacher's turn to be puzzled! It took a while for the writer to explain to him that the two units were equivalent.

(3). Non-development of a repertoire of basic, yet essential mathematical skills and facts.

This is undoubtedly a serious impediment for further learning of mathematics. When the basic facts or skills needed for a task are not ready at hand, they have to be acquired first. This is both time consuming and less economical in terms of effort involved. The problem is aggravated owing to a wide gap between procedural skills and the conceptual understanding of those procedures (Wood, 1988).

(4). Non-development of an adequate level of language facility.

In particular, limitations in comprehension often led to the use of inappropriate algorithms. It also seemed to thwart the development of spatial ability and powers of visual imagery. The need for "a balance of sensitivity to visual imagery with language"(Dawe, 1984) was indicated here. Also, a wide gap seemed to exist between ordinary language and mathematical language. The importance of student "talking and writing" on mathematical topics for the development of conceptual knowlewdge in mathematics has been highlighted by many writers in recent times. (See, for example, Davis, 1989; MacGregor, 1990; Miller, 1991). Few teachers, however, seem to have realised the usefulness of language as a strategy for teaching mathematics (Swinson, 1992).

SUGGESTED PROCEDURES FOR REMEDIATION

The writer strongly feels that a programme of action for remediation must aim at the following:

(1). The building up of a repertoire of essential basic mathematical concepts as a springboard for the proper acquisition of further conceptual knowledge.

(2). The development of a repertoire of essential basic mathematical facts and skills as necessary tools for future learning.

(3). The development of language ability, with progressive narrowing down of the gap between ordinary language and mathematical language.

The writer also feels that action for remediation must be built into the university courses as integral components thereof, rather than separate programmes. Thus it would be a continuous and an on-going programme of action. It is no use pinning the blame on schools for student lapses and leaving the students to fend for themselves.

A remedial programme should include

(a) recapitulation of essential background knowledge through appropriate questioning.

(b) an introductory "probe" to serve as a diagnostic procedure before embarking on a new topic, especially a hard one.

(c) a quick but effective revision of relevant background material, presumed to have been learnt at school. This would enable students to dispel any erroneous misconceptions held by them.

(d) placing great emphasis on conceptual development at all levels of teaching.

(e) reinforcing conceptual knowledge by abstraction and generalisation through intensive drill.

(f) emphasizing the importance of language at all levels and verbalising mathematical thought as often as possible, with a view to narrowing the gap between ordinary language and mathematical language.

(g) presenting material in such a manner as to bring out the underlying structure and the unity between such structures wherever it exists.

(h) placing great emphasis on orderly, logical and accurate presentation of pupil work.

It must be emphasized that most of the above procedures would entail a radical re-orientation of the traditional "lecture type" of teaching of most courses at the university. Furthermore, it would entail the revision of most existing courses in order to accommodate the measures proposed.

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