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The Influence of Visual Cues in Interpreting 3-D Diagrams

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Note: Graphics can be found at the end of this article.

Abstract: The purpose of this study was to explore the nature of the visual cues used by learners of mathematics in South African Schools. Visual cues were defined as part of the students mental processes and not as an element of the diagram. Participants between the ages of 12 and 18 were interviewed using three dimensional diagrams that may be experienced in the mathematics classroom. Four visual cues used by the students were indentified. Two of the influences of the use of these cues on the spatial perception of students are discussed.

In the learning of mathematics the student will at some point be expected to interpret three dimensional diagrams. There is however often an assumption made by mathematics educators that students have the necessary visualisation ability and experience with the conventions of three dimensional diagrams to appropriately interpret diagrams presented to them in the classroom (Bishop, 1986). The visual-pictorial ability of children is distinct from the verbal-logical ability (McGee, 1979; Battista, 1990) and therefore requires particular attention in our attempts as mathematics educators to understand the spatial reasoning of our students.

I will refer to a two dimensional representation of three dimensional space based on the spatial conventions of society as a three dimensionl diagram. Spatial conventions have been established by society as a means of ordering spatial communication and therefore `The tradition of representing three dimensions in two has the characteristic of an arbitary linguistic convention' (Segall, Campbell & Herskovits, 1966, p.94). Three dimensional diagrams are considered distant representations of space because they are dimensionally inferior to the space they represent and are therefore particularly subject to the conventions of representation (Parzysz, 1988).

The conformity of the representer to spatial conventions of society as well as the interpreter's knowledge of those conventions are necessary for effective visual communication (Parzysz, 1988). However to facilitate the mathematics learner appropriately perceiving three dimensional diagrams a knowledge of spatial conventions is not sufficient. Bishop Berkeley claims that `we identify them (pictures) and act not so much according to what is directly sensed but to what is believed' (Gregory, 1970, p.11). The student's `believed' perception of a three dimensional diagram is determined by a mental construction of the image rather than the properties of the diagram. The diagrams are perceived through the `lenses of our past', making spatial perception as much a cultural phenomenon as mathematics (Gregory, 1970; Bishop, 1979). Consequently the perception of diagrams is influenced by both the psychological and the socio-cultural factors of the interpreter's experience (Cohen & Akarsu, 1991; Jahoda, 1980; Shar & Geeslin, 1980).

Students who are familiar with the conventions of the representer may not perceive the diagram in the way intended by the representer (Parzysz, 1991). The perception of diagrams using familiar conventions may not result in the visualisation of the diagram but rather as Piaget suggests a reproductive knowing of what the diagram represents (Clements, 1981; Parzysz, 1988). However, when the diagram or spatial convention is unfamiliar the interpreter is required to construct an anticipatory mental image that may well be more vivid than the reproduced knowing of a diagram (Clements, 1981; Parzysz, 1988). Therefore the concepts of visual imagery and perception, although both pertinent to the study of visual-pictorial ability, cannot be conflated and it is the later that is the focus of this paper.

Authors in the field of spatial perception suggest that the perception of the student is influenced by `cues` contained in the diagram as well as previous experience with the conventions of the representer (Deregowski, 1980; Segall et al, 1966). I argue that it is only the conventions that are contained in the diagram and that the cues are in fact part of the mental processes of the student. Cues are not core aspects of the diagram for this would be a duplication of the role of spatial conventions. The cues act as initiators of the mental images and are therefore mental processes that influence how children perceive the conventions illustrated in a diagram. I would therefore call those aspects of the diagram that children focus their attention on the dominant convention which is only dominant because of the visual cue used by the student.

The use of suitable visual cues by the student determines their interpretation of the spatial conventions and forms the basis on which the perception of a diagram is established. Furthermore it is the perceived nature of the diagram not the mental image that influences the actions of the student on the diagram or as a result of the diagram in a problem solving situation.

The perception of diagrams and visual imagery of students is of an individual nature (Bishop, 1989). The identification of visual cues used by students in their perception of diagrams does not detract from the individual nature of perception. Rather it acts as a tool for mathematics educators in their efforts to understand the nature of students' spatial perception.

The purpose of this paper is to identify the nature of the visual cues used by learners of mathematics in their perception of three dimensional diagrams. I will firstly discuss each of the four visual cues used by the students in their interpretation of the diagrams presented to them. Secondly, I will discuss the influence of the cues on the perception of the student and the resulting actions that the student takes when solving a problem directly related to the diagram. These discussions will be illustrated by protocols.

RESEARCH METHODS

Research methodology in the field of spatial ability is as much debated as the subject itself (Clements, 1981; Bishop, 1980). I have used verbal self-reporting as the primary source of information in my attempt to understand the spatial perception of children. The appropriateness of self-reported mental activity as a research method in the study of spatial ability has been questioned by Nisbet and Lohman (Clements, 1981). However verbal reports have been used by Ericsson and Simon as well as Kosslyn as a meaningful research method in the understanding of children's thought processes (Clement, 1981). Self-reporting is only suitable as a research method when it occurs after the task has been completed rather than concurrently. The research method I have employed is suitable for two reasons. Firstly, the verbal self reports where only undertaken after the task had been completed. Secondly, the research has as its primary focus the visual cues that students use in spatial perception rather than the spatial visualisation of the students. The research method assumes that the actions taken by children are always rational given their understanding of the task and their intentions.

Participants

Four groups of South African high school students where chosen to participate in the initial research. The groups were chosen on the basis of age and socio-cultural background.

Students from different socio-cultural backgrounds participated in this research. This included students from a Western culture as well as an African culture, attending state schools. The participants are from upper and lower middle income urban communities. The purpose of selecting students from different socio-cultural backgrounds is too facilitate a broader research perspective of students' spatial perceptions in South Africa and not to undertake a socio-cultural comparison.

All the students had attained average mathematics results based on their scholastic achievement. Participants were of the 12-14 age group in their eighth year of schooling and the 15-18 age group in their tenth year of schooling. The purpose of choosing participants from these age groups is the levels of spatial perception and visualisation that is required in the learning of mathematics during these years of schooling in South Africa.

Initial Testing

An initial test has been given to each group of students in a classroom setting. Each of the groups given the test knew either that I was a mathematics teacher or they were given the test during the mathmematics lesson of that day. The initial test consists of 3-D diagrams in parallel perspecive that require the student to answer specific questions in a written format. The research of Parzysz (1991) suggests that students are most comfortable when viewing three dimensional diagrams in parallel perspective. Parallel perspective is also the most commonly used in South African mathematics textbooks. The diagrams require the use of spatial orientation or visualisation as well as a knowledge of spatial conventions. The purpose of the test was originally to act as the basis on which the interviewees were choosen. Although it fulfilled this role it also become part of the interviews and yielded interesting insights into the students spatial ability.

Interviews

In the interview the students were given a diagram sheet that required a written description of their perception of the diagrams. The interviews were initiated by recalling these descriptions. Eight students were interviewed, two from each group of participants. The purpose of the interview was to elaborate on how the student comes to perceive in the way they do rather than an elaboration of what the student perceived. I was primarily interested in the child's perception and not in the ability to perform mental manipulations of their images. A translator was present during my interviews with the 15-18 year old second language speakers. There was however no translator present during my interviews with the 12-14 year old second language speakers. I was informed that these students were sufficiently proficient in the English language and that an interpreter was not necessary.

VISUAL CUES USED BY MATHEMATICS LEARNERS

Three dimensional diagrams experienced by a student in the mathematics classroom are not perceived on the basis of spatial conventions contained in the diagram but rather on the basis of the visual cues used by the student in the interpretation of the diagram. I must stress that the visual cues used by the interpreter are not contained in the diagram but form part of the mental visualisation process of the interpreter. A suitably drawn diagram may facilitate the use of an appropriate visual cue, however the use of a visual cue cannot be determined by the diagram. The nature of the visual cue used by the interpreter influences how the interpreter perceives the diagram.

Through the analysis of the protocols, four visual cues used by the students, in their interpretation of the diagrams presented to them in the interviews, have been identified. These are spatial cues, environmental cues, geommetrical cues and artistic cues. The use of a particular visual cue does not appear to be persistent and thus given the same diagram at a later time the perceived image may well vary. It is however not the purpose of this paper to consider what factors influence the choice of a cue when the student is exposed to a diagram. The student's use of these cues and the influence of the cues on the subsequent actions of the student will be discussed below.

Spatial cues

A spatial cue is the use of the knowledge of a spatial convention in the interpretation of a diagram. Thus the effective use of a spatial cue is only possible when the student has a knowledge of the spatial conventions used by the representer. A spatial cue results in the student focussing their attention on a specific spatial convention in the diagram and interpreting the diagram on the basis of their experience with the convention. The resulting perception of the diagram is then determined by the spatial cue used by the student.

(See end of article for figures.)

Figure 1. How many sides does this object have?

100 I: How did you get 12?

101 D: There is 1, 2, 3, 4, 5 and then 6, and a 7, 8, 9, 10, 11, 12. (Counted shapes enclosed by solid lines as well as broken lines)
102 I: Right. Why do you count these dotted lines?
103 D: Because I know the other sides.
....
109 I: Have you seen dotted lines before?
110 D: Yes

111 I: What does a dotted line usually mean to you?
112 D: To show the other lines that you can't see.

Delon, 13, focussed his attention on the dotted lines (line 101) resulting in him counting 12 faces rather than counting six faces and then multiplying by two. Delon's use of a spatial cue is illustrated by his use of the dotted lines (line 112) in interpreting the diagram (figure 1). The use of the spatial cue has focussed his attention on the spatial conventions used in the diagram.

The value of using a spatial cue is determined by the interpreter's knowledge of the conventions of diagrammatic representations. This does not imply that those students who lack experience of spatial conventions do not use spatial cues to interpret diagrams but rather that the usefulness of the spatial cue is restricted and will often result in miscommunication. This is illustrated by the following protocol:

Figure 2. How many small boxes are there in the large box?

Large box

Small box

7 I: How many sides does this small box have?

8 T: Three sides.
9 I: How many sides does the big box have
10 T: Three
11 I: Also three. Think of a picture in your head, that picture of a box.
12 T: There's four sides
13 I: There's four sides. Show me where the the four sides are.
14 T: The other one is here.

Taphelo, 12, has used a spatial cue in her interpretation of the diagram (figure 2). This is illustrated in her written response where she interprets the diagram as a box. She is unable to restore the full meaning of the diagram because of her limited experience with the spatial conventions used in three dimensional diagrams. Thus although she uses a spatial cue to interpret the diagram, the spatial cue results in a perception that the number of sides of a cube are only three (line 8,10). Taphelo interestingly sees four sides (line 12) after being prompted by the interviewer (line 11) but is unable to extend the principle of unseen sides to include all six sides of the box.

Environmental cues

An environmental cue results in the interpretation of the diagram as a representation of some experienced object. As will be illustrated in the following protocol the object has usually been previously experienced by the interpreter. The attention to spatial conventions is replaced by a focussing on the environmental aspects of the diagram. Thus the diagram is perceived as a previously experienced object rather than a mathematical representation of space.

159 I: And this one here, how do you get twelve?
160 R: I said 1, 2, 3, 4, 5, 6. I said six then it's like, then let me make an example with a soccer ball. I turn it around to see what's on the other side and I see the same blocks. Say it's timesed 2 it gives me 12.

Raymond, 17, established a three dimensional interpretation of the diagram (figure 1) by means of an environmental cue that is based on his experience of a soccer ball (line 160). His visual image of a soccer ball along with arithmetic manipulation allows him to succesfully determine the number of sides represented in the diagram (line 160). The environmental cue used by Raymond allowed him a useful means of appropriately interpreting the diagram as a three dimensional representation thus compensating for his lack of experience with the spatial apsects of a dodecahedron. The usefulness of environmental cues in the interpretation of three dimensional diagrams will be discussed later in the paper.

Geometrical cues

A geometrical cue is the use of the knowledge of a geometrical label and results in the interpreter perceiving the diagram as a mathematical representation. This is displayed by the use of geometrical nouns or adjectives in the verbal description of the student's interpretation of the diagram. The use of geometrical cues is directly related to the experience of the student with formal geometries. The use of geometrical cues was substantially less commonplace than the use of both environmental or spatial cues. The discrepancy in the use of cues is accentuated by the fact that the participants were all aware that the research was of a mathematical nature. The limited geometry experience of South African students may account for this discrepancy.

Figure 3. What do you see in the picture?

209 I: In this picture here what do you see?
210 W: A square
211 I: A square. Just one square?
212 W: No
213 I: How many?
214 W: Three
215 I: Show them to me.
216 W: This one, this one, this one.

Wellington, 18, displayed the use of a geometrical cue in his interpretation of the diagram (figure 3) by the use of the term square (line 210). Square is used as noun rather than an adjective illustrating that he perceives it as a square rather than as a square like shape. He had interpreted the diagram as a two dimensional representation on the basis of his geometry experience. The lack of experience with three dimensional geometries as well as Wellington's use of a geometrical cue results in the two dimensional interpretation. Considering that figure three is usually used by mathematics educators to illustrate a cube, Wellington's interpretation would result in a miscommunication of ideas.

Artistic cues

An artistic cue is the use of an artistic perspective in the interpretation of the diagram. The use of an artistic cue results in the attention of the student being focussed on the aesthetic appearance of the diagram. Artistic cues were not prevelant in the research. This may be due to the environment in which the tests and interviews were undertaken.

Figure 4. Circle any mistake in the picture (If you think there is a mistake).

149 I: Why did you say that's wrong?
150 H: Because this one is a little bit shorter than that door and you can't see out of the windows.
151 I: Alright, so then, what's wrong with these windows?
152 H: They're to high.
153 I: Okay and is there anything else wrong with that picture?
154 H: Not that I can see.

Heidi, 15, focusses her attention on the artistic nature of the diagram (figure 4) rather than the breach of spatial convention. Her use of the artistic cue is illustrated by her concern that the windows have been drawn to high for people to look out of them (line 150). Her ability to interpret spatial conventions is displayed on many occasions during the interview. I suggest that she is not unable to see the error (line 154) but rather that she ignores the window on the side of the house that has been drawn out of perspective because of her use of an artistic cue and not a spatial cue.

THE INFLUENCE OF VISUAL CUES ON THE SPATIAL PERCEPTION OF LEARNERS

As a mathematics educator I place demands on the visual ability of the learners of mathematics. Thus there is a need to be aware of the interpretations that the learners may make of the diagrams. It is for this reason that I consider the influence of visual cues on the spatial perception of the learners of mathematics.

It is the student's perception of the diagram that will determine how they respond to diagrammatic representation presented to them. The use of a particular visual cue influences the interpretation of the student which in turn influences the actions of the student.

I will consider two of the influences that were apparent during the research. Firstly, the role played by the visual cues in influencing three dimensional as opposed to two dimensional interpretation of diagrams. Secondly, the influence of visual cues on the choosing of an algebraic as opposed to a visual method of solving the problem.

Two dimensional versus three dimensional perception

Generally when the participants used geometrical cues in the interpretation of the diagrams they perceived the diagram as a two dimensional figure. The perception resulting from a geometrical cue may be influenced by the South African mathematics curricula which includes very little three dimensional geometry and thus the students are perceiving two dimensional images when using a geometrical cue. In contrast when the participants used environmental cues in the interpretation of the diagram they perceived the diagram as a three dimensional figure or image. The usefulness of an environmental cue is corroborated by Deregowski (1976) when he suggests that the contextualisation of the diagram may be a useful means to establish depth perception.

This can be observed in the protocols of Raymond and Wellington mentioned earlier in the paper. I have chosen to illustrate the point with a protocol taken from an interview with the same participant.

Figure 5. What do you see in the picture?

210 I: Okay this here, you say you see a page. Describe to me how you see a page.
211 R: Because its something which is drawn down with four corners and its straight. That's why I thought it's a page.
212 I: Okay, describe to me more about this page. How is it lying? What is it doing?
213 R: Actually I could say it's not really a page but that's what I thought it's a page because it's not so straight and it's, how can I say, just like the way they draw it, it's lying down.

Raymond has used an environmental cue in his perceiving the diagram (figure 5) to represent a page (line 210). The geometrical interpretation of the diagram is a trapezium, a shape that should be familiar to Raymond. The diagram includes no conventions that would suggest that it is a three dimensional representation and yet Raymond perceives it in three dimensions (lines 213). In contrast Raymond perceives figure six in two dimensions.

<u>Figure 6</u>. What do you see in the picture?

225 I: You say you see a triangle. Describe to me how you see a triangle.
226 R: As I know it , it's because it has three corners.
227 I: How many triangles are there?
228 R: It's three triangles.
229 I: Three triangles. How do you see the triangles?
230 R: Ja, because I can see them the way they draw it and they put them together to form one triangle. I can differentiate by showing it's like this one, it's a triangle and this one and this one.

Raymond has used a geometrical cue in perceiving the diagram (figure 6) as a set of three triangles in two dimensions (lines 225,226). The diagram is more often used to represent a tetrahedron, a shape that many students call a pyramid. I do not know whether Raymond was familiar with this representation but would suggest from the complete protocol that he was probarbly unfamiliar with the diagram.

It is noticeable that the use of the environmental cue resulted in a three dimensional perception when the diagram (figure 5) was least likely to facilitate such a perception. In contrast a diagram (figure 6) that may facilitate a three dimensional perception was perceived in two dimensions when using a geometrical cue.

Furthermore this instance would substantiate the notion that the cue used by the student is situated in the mental processes and not in the diagram. Raymond,s perception of figure five as a page cannot be attributed to a "cue" in the diagram but must be as a result of his mental processes perceiving the diagram on the basis of experience as well as a visual cue that is within the mental process.

<u>Algebraic versus Visual means of solution</u>

The use of spatial cues resulted in different methods of solving spatialy presented problems. The participants used both algebraic as well as visual methods to solve the problem illustrated in figure two. There appeared no consistency in the use of the spatial cue as a factor resulting in the use of algebraic as opposed to visual methods of solving graphically presented problems.

10 W: I got nine boxes, small boxes. Then on that other side, I've got nine boxes. So if I calculate all the big box with other boxes which are not drawn, I got ..

11 I: Which are?
12 W: Are not drawn.
13 I: Where are they if they're not drawn?
14 W: Um.. they're not drawn because you cannot see through the paper there.
15 I: Oh I see. So are they behind?
16 W: Yes they are behind. So I calculate them all, I got 9, 18, 27. I got 27 then I added them twice, 27 and 27 and I got 54.
17 I: Right. How many sides does this little box have?
18 W: It has six.

Wellington is using a spatial cue in his perceiving of the diagram (figure 2). He perceives the diagram as a three dimensional box (line 10) and displays a knowledge of the spatial conventions by identifing the existence of the rear of the box (line 12,18). The use of the spatial cue is not accompanied by spatial orientation that would allow him to perceive the representation as a large box constructed of 27 small boxes as was intended by the representer and therefore the value of using the spatial cue is limited. The use of the spatial cue results in a method of solution for which Wellington does not have sufficient spatial orientation to make the method viable and as a result calculates 54 small boxes in the large box (lines 16). The use of a spatial cue may not result in the use of visual methods of solution.

1 I: How do you get that there are 27?
2 A: I times the length by the breadth and then times by the height.
......
23 I: So how many is that, that you can see?
24 A: ..(24 seconds).. 21
25 I: 21, alright. Then how many can't you see?
26 A: ..(10 seconds).. Okay there's four at the bottom on the other side and then there'll be another four on top of those so it's eight.

Ansel, 16, has used a spatial cue in his perception of the diagram (figure 2). This is displayed by his perceiving the diagram as a cube and hence his interpretation of the sides as the length, breadth and height (line 2). The spatial cue has resulted in the use of an algebraic method of solution where he uses the formula for volume to determine the number of small cubes in the large cube (line 2).

Ansel displays a flexible thought pattern in that he is able to spatially orientate the diagram to determine the number of hidden cubes (lines 26). Surprisingly, he does not use this spatial ability to solve the problem but has rather used an algebraic method of solution. Ansel was one of the few students who was able to determine the number of hidden cubes and yet he is the only student who algebraically solved to this problem.

What determines the students use of a visual or algebraic method of solution is unclear from this research. Presmeg (1986) showed that South African students tend not to use visual methods of solution but usually choose to solve problems algebraically. That Ansel, who can be considered spatially able, should decide to use an algebraic solution suggests that the students process of selecting a method of solving spatially orientated problems requires further research.

CONCLUSION

Four visual cues used by students of mathematics have been identified as pertinent to the student's spatial perception. These visual cues I have argued are not elements of the diagram but rather part of the mental processes of the student. The exact nature of the cue as a mental process cannot be explained on the basis of this research and requires further study.

An understanding of the visual cues is useful because it allows us to (1) be more cautious in our representation of diagrams and expectations of students' perceptions; (2) facilitate the use of effective visual cues to alleviate spatial perception difficulties students may encounter. This was illustrated by the effectiveness of environmental cues in perceiving three dimensions from a two dimensional representation. I would argue that the identification of these cues is not only useful in our understanding of students' perception of mathematical diagrams but also students' perception of diagrams presented to them in other learning environments.

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Figure 1. How many sides does this object have?

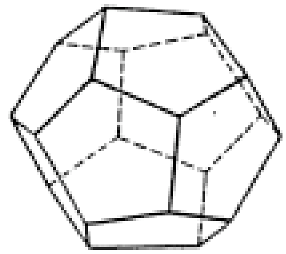

Figure 2. How many small boxes are there in the large box?

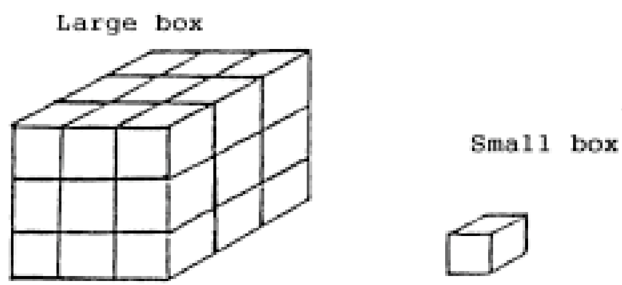

Figure 3. What do you see in the picture?

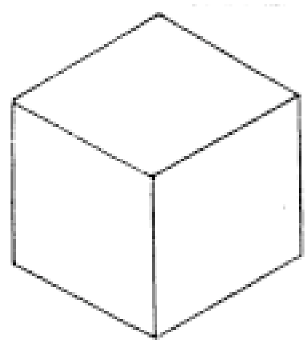

Figure 4. Circle any mistake in the picture (If you think there is a mistake).

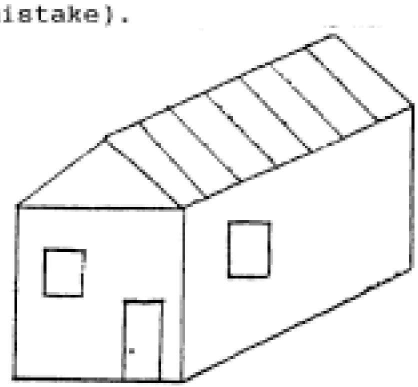

Figure 5. What do you see in the picture?

Figure 6. What do you see in the picture?