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Paper Title: MISCONCEPTIONS IN SPACE - TIME GRAPHS
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Abstract: The world of mathematics is an ideal world : points, lines, planes have no thickness; it is possible to see three objects but no one has ever met the number . This explains why representations are so important in mathematics. These representations rest upon conventions : e.g. in cartesian graphs, the dots we draw represent mathematical points, with no thickness, and two different dots always represent two different points. Usually, as far as pure mathematics (algebra, analysis) are concerned, these conventions are fairly clear for the students.

As this paper shows, problems arise when mathematical tools are used to represent concrete situations. The representations look more or less the same as the ones used in pure mathematics, but the conventions used are sometimes very different. In space - time graphs, the dots used do not represent a mathematical point : they represent a <u>place</u> - e.g. the station in a railroad graph - with some area, (otherwise, how could two trains cross each other or overtake each other without crashing). Moreover, two different dots can represent the same place, at different times. In geographic maps, the situation is again different : a dot can represent a whole town ! The differences in representations are seldomely made clear to the students, in Belgium at least. Very often, the conventions are used implicitely by the teacher.

The preliminary experiment described below shows that these implicit conventions are not necessarely understood by 10-graders, even if they are considered as high achievers in mathematics, nor by elementary school students teachers.

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MISCONCEPTIONS IN SPACE - TIME GRAPHS

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INTRODUCTION

The world of mathematics is an ideal world : points, lines, planes have no thickness; it is possible to see three objects but no one has ever met the number . This explains why representations are so important in mathematics. These representations rest upon conventions : e.g. in cartesian graphs, the dots we draw represent mathematical points, with no thickness, and two different dots always represent two different points. Usually, as far as pure mathematics (algebra, analysis) are concerned, these conventions are fairly clear for the students.

As this paper shows, problems arise when mathematical tools are used to represent concrete situations. The representations look more or less the same as the ones used in pure mathematics, but the conventions used are sometimes very different. In space - time graphs, the dots used do not represent a mathematical point : they represent a place - e.g. the station in a railroad graph - with some area, (otherwise, how could two trains cross each other or overtake each other without crashing). Moreover, two different dots can represent the same place, at different times. In geographic maps, the situation is again different : a dot can represent a whole town ! The differences in representations are seldomely made clear to the students, in Belgium at least. Very often, the conventions are used implicitely by the teacher.

The preliminary experiment described below shows that these implicit conventions are not necessarely understood by 10-graders, even if they are considered as high achievers in mathematics, nor by elementary school students teachers.

DESCRIPTION OF THE EXPERIMENT

Population :

Group 1 : 23 10-graders, majoring in mathematics (6 hours of mathematics a week).

Most of them are from a low socio-economical background (children of fairly new immigrants). Only three of them are native french speakers.

Group 2 : 14 students of an elementary teacher-training college (year before graduation). Most of them are native french speakers.

SETTING

Cartesian graphs and space - time graphs are introduced in Belgium as early as grade 5. Space-time graphs are extensively used in physics class in grade 9.

The following questionnaire - taken from a grade 5 textbook - has been presented to the 10-graders at the end of a chapter on linear functions, and to the student teachers at the end of a chapter on problem solving techniques. The students had 45 minutes to answer the questionnaire.

Answer the following questions by TRUE, FALSE - or ? (Impossible to tell from the graph). Explain your answer in a few words.

a) The school-janitor is walking.

b) Peter overtakes Ann and then waits for her.

c) At a certain moment, Ann gets on the back of Peter's bike.

d) Peter gets off his bike and walks next to Ann.

e) Ann and Peter are siblings.

f) The way from home to school is uphill.

Answer the following questions :

g) Did Ann pass the janitor during her trip from home to school ? If yes, how many times ? (beware : to pass and to overtake are two different ideas).

h) How many kilometers did the janitor cover altogether ?

i) What is each one doing at 7h.40 (place and occupation) ?

j) Who is the fastest ? Why ?

(For a solution of this questionnaire, see annex)

RESULTS : GENERAL

For most questions the ratio of correct answers is about the same in both groups. Nevertheless, the 10-graders rank better for questions c and d while the student teachers rank better for questions f and i.

Within each groups, there seems to be no relation between and .

The last question is the only one correctly answered by a majority of students (19 out of 23 in group 1; 13 out of 14 in group 2).

The answers show that the mistakes can be sorted in four groups :

1) The student does not understand the specific conventions of space - time graphs (e.g. he/she claims that Peter and Ann do not live at the same place) or he/she is fooled by the appearance of the graph (e.g. he/she claims that the way from home to school is uphill, by); he/she does not understand that if the two lines describing the journeys cross, people meet or overtake each other nor that a line parallel to the x - axis means that the mobile doesn't move.
2) The student does not make use of all the information one can get from a space - time graph, especially as far as speed is concerned. (e.g. he/she does not realise that, after 7h.45 Ann and Peter travel at the same speed).
3) There is a language problem : it seems that the word is not understood correctly.

4) Psychological elements interfere (e.g.).

Very few questions are left unanswered or unexplained.

RESULTS : DETAILED

GROUP 1 : 10-GRADERS (23 students)

Question	Correct answers	Mistakes
a	14	Type 1 : (2)[1] Type 2 : (7)
b	15	Type 1 : (1) (2) (3) Type 2 : (3) Type 3 : Confusion to overtake - to pass (1) (1)
* [2]		
c	7	Type 1 : parallel » (3) Type 2 : (12) Type 3 : — — Type 4 : — — Unanswered or unexplain (1)
d	7	Type 1 : parallel » (1) Type 2 : (15)
e	4	Type 1 : (7) neighbours » (2) village » (1) Type 2 : — — Type 3 : — — Type 4 : (5)
* [3]		Unanswered or unexplained (5)

[1] Number of answers of this type.
[2] Several subjects gave more than one answer.
[3] One subject gave two answers.

Question	Correct answers	Mistakes
f	11	Type 1 : can see that from looking at the graph » (5) Type 2 : — — Type 3 : — — Type 4 : the way went uphill, they would go slower and slower, especially Ann » (4) Unanswered or unexplained (3)
g *[4]	1	Type 1 : one South » (1) (1) (1) Type 2 : (1) Type 3 : (17) Type 4 : — — Unanswered or unexplained (2)
h	12	Type 1 : (7) Type 2 : that the distance is about 7 km » (2) Type 3 : (1) Type 4 : — — Unanswered or unexplained (1)
i *[5]	5	Type 1 : (7) (1) Mistakes in reading the distances (4) Type 2 : — — Type 3 : Question answered partially (either place or occupation is stated) (7) Type 4 : — — Unanswered or unexplained (1)
j	19	Type 1 : (1) Type 2 : (1) Type 3 : — — Type 4 : — — Unanswered or unexplained (2)

[4] One subject gave two answers.
[5] Two subjects gave two answers.

GROUP 2 : STUDENT TEACHERS (14 students)

Question	Correct answers	Mistakes
a	8	Type 1 : — — Type 2 : (6)[6]
b * [7]	8	Type 1 : (1) (5) Type 2 : — — Type 3 : — — Type 4 : know each other » (1)
c * [8]	3	Type 1 : parallel » (1) Ann got on Peter's bike her line would becomea dotted line after 7h.50 » (1) Type 2 : (10) Type 3 : — — Type 4 : he do so the first time they met » (1)
d * 7	3	Type 1 : (1) (1) (1) Type 2 : (8) Type 3 : — — — Type 4 : do so the first time they met » (1)

[6] Number of answers of this type
[7] One subject gave two answers.
[8] Several subjects gave more than one answer

Question	Correct answers	Mistakes
e * 8	3 * [9]	Type 1 : (2) (1) [10] (2) Type 2 : — — Type 3 : — — Type 4 : (2) Unanswered or unexplained
f	13 * [11]	Type 1 : (1)
g	1	Type 1 : (2) (1) Type 2 : — — — Type 3 : (10)
h	8	Type 1 : (2) (2) [12] (1) Type 2 : — — — Type 3 : — — — Type 4 : — — — Unanswered or unexplained (1)
i	5	Type 1 : (3) (1) (1) Type 2 : — — — Type 3 : occupation is stated » (4)
j	13	Type 1 : (1)

[9] Among these three, one answer reads :

[10] Theoretically they could be neighbours. The answer would have been accepted if the students had been more precise e.g. .

[11] One answer reads :

[12] Wrong unit

DISCUSSION

Space - time graphs use conventions that are fairly different from the ones used in cartesian graphs :

- Space is one - dimensional : all mobiles travel the same route.
- One line represents one mobile; two different mobiles are represented by two lines, sometimes different in graphism.
- The dots in a space - time graph represent places, with a certain area. Howewer this is a . It varies from graph to graph or even within the same graph.

  In our test, for instance, the dots for the school or the home represent one building; an unspecified dot in the graph represents a spot in the street at a certain time. In a railroad graph, a dot for a town represents a station in this town.
- Two different dots can represent the same spot, at a different time.

From these conventions, one can deduce a.o. :

- If the line representing a mobile is parallel to the x - axis (time - axis), the mobile stays at the same place for a certain time.
- A space - time graph gives no information about the geographic features; it states only where the mobiles are at what time, and thus gives information about their speed.

The experiment described above shows that these conventions are not clearly understood by 10-graders, even if they are considered as high-achievers in mathematics, nor by elementary school student teachers.

We claim that most mathematics and physics teachers are unaware of this; questions usually asked for a test on these topics are indeed asked in the language of the space - time graphs themselves -**object-language**- and these questions are usually answered correctly (e.g. question i :).

The misconsceptions become obvious when students are given an opportunity to express themselves **about** the concrete situation the space - time graph is supposed to represent (**meta -language** of the representation).

CONCLUSION

The fact that students and teachers unknowingly do not use the same conventions when dealing with a given type of representations (here space - time graphs) seems to be a new example of the language gap described by Lowenthal (1987).

Lowenthal insists on the ambiguities caused by a difference in the mastery of verbal language between the young child and the adult.

The gap we observe here seems to be of a similar nature : the learner and the specialist do not have the same perception of the representation system they use.

In both cases, both parties remain unaware of the misunderstanding between them, as long as their interactions and exchanges of ideas remain within the scope of the object-language of the representation : the misunderstanding rests upon misconceptions at the meta-level.

Lowenthal (1992) claims that the language gap can be clarified by using another representation, provided with strong technical constraints that make the logical structure of the language clear, and thus reveal the .

We think that a test like the one presented in this paper, since it deals with the meta-level of the representation, can help to clarify the perception of space - time graphs, **provided a correction session is planned**, where the **conventions are explicitely stated** and a limited number of elements (points) are added to the graph in order to concretise it.

The first steps on such a research have been undertaken ; the results obtained until now seem promising.

ANNEX :

Answer the following questions by TRUE - FALSE - or ? (Impossible to tell from the graph). Explain your answer in a few words.

a) The school-janitor is walking. **FALSE**
   He covers 2,5 km in 5 min. i.e. 30 km/h
b) Peter overtakes Ann and then waits for her. **TRUE**

   Peter overtakes Ann a little later than 7h.35, and waits for her from 7h40 to 7h.45.
c) At a certain moment, Ann gets on the back of Peter's bike. **TRUE**
   After 7h.50 they travel together, and their speed is the same as Peter's, on

his bike.

d) Peter gets off his bike and walks next to Ann. **FALSE**
See c).

e) Ann and Peter are siblings. **? or TRUE**

They live in the same house.

f) The way from home to school is uphill. **?**

The graph does not give that kind of information.

Answer the following questions :

g) Did Ann pass the janitor during her trip from home to school ? If yes, how many times ? (beware : to pass and to overtake are two different ideas).

Ann passed the janitor once, a little before 7h.30.

h) How many kilometers did the janitor cover altogether ?

5 km.

i) What is each one doing at 7h.40 (place and occupation) ?
The janitor leaves the newspaper stand, and starts his way back to school.
Ann is half-way to school; she is walking.
Peter is 2 km from home; he starts waiting for Ann.

j) Who is the fastest ? Why ?
The school-janitor. He covers 2,5 km in 5 min; Ann covers less than 0,5 km in the same time and Peter less than 2 km.

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