From Misconceptions to Constructed Understanding
The Fourth International Seminar on Misconceptions Research (1997)

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Author: Masters, Jenny

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It was found that the computer-based investigation proved to be a nourishing environment for application of existing mathematical understandings and the construction of new ideas. The children participating in this study demonstrated many mathematical “moments” and post tests revealed considerable improvement in geometric understanding. This success however, was seen to rely on the supportive role played by the researcher. The project report proposed that the results of this type of computer-based project would be highly dependent on teacher-support mechanisms such as scaffolding.

Further research is warranted to examine the nature of teacher support and intervention during on-computer exploration tasks. It is suggested that this scaffolding should be characterised in to categories such as technical, social, emotional and cognitive. Additionally, it is deemed important that teachers are aware of these scaffolding processes and can determine when it is appropriate to intervene in the exploration process and when scaffolding should be withheld. It is anticipated that understandings of this nature will lead to richer learning experiences for young children exploring with technology-based tasks.

Keywords: geometry, technology, scaffolding
General School Subject:
Specific School Subject:
Students:

Macintosh File Name: Masters-Geometry
Release Date: 10-2-97 A, 10-23-97 B

Editor: Abrams, Robert

Publisher: The Meaningful Learning Research Group
Publisher Location: Santa Cruz, CA
Volume Name: The Proceedings of the Fourth International Misconceptions Seminar - From Misconceptions to Constructed Understanding
Publication Year: 1997
Conference Date: June 13-15, 1997
Contact Information (correct as of 12-23-2010):
Web: www.mlrg.org
Email: info@mlrg.org

Investigations in Geometric Thinking: Young Children Learning with Technology

Jenny Masters
The RITE Group
Queensland University of Technology

Abstract

This paper describes a research project based on the implementation of a computer-based mathematics curriculum, characterised by tasks designed to promote exploration and investigation of geometric concepts.

The project investigated the domain of early mathematical learning and application, with an emphasis on the development of spatial sense and geometric knowledge. It also it examined a learning context that encouraged children's active construction of mathematical meaning, rather than the passive transmission of facts. Additionally, it considered the implications of utilising computers as a dynamic learning environment.

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Computers in Curriculum

Whereas the notion that computers could be utilised by children was considered truly revolutionary in the 1970's, today the idea of computers “belonging” to children is widely accepted (Miller 1994). Computers play an integral role in our society and as children are exposed to computer applications in their daily lives, so they adopt the technology a tool for their own purposes. The implementation of technology into education however, has been less revolutionary. As technology has changed other disciplines such as communication, medicine, engineering and even entertainment, the adoption of technology in the educational context has been less than inspiring and remained static over the last decade (Papert 1993). While computers are beginning to appear as standard items in the classroom scenario, the use of this equipment is often mundane; the computers are merely used to reinforce existing educational practices rather than as a catalyst for educational innovation.

Research has illustrated that the most frequent consequence of the introduction of computers in schools is the “drill and practice” game (Hickey, 1993; Clements 1994; Becker 1992; Collis 1988). Such software applications reinforce basic skills in content areas such as mathematics operations or phonetic blends and are most frequently used by teachers as a reward or to add variety to diet of skill practice. While there is some evidence that these programs can promote basic skills (Kelman in Clements, Nastasi and Swaminthan 1993), the educational value of such applications is limited. There may also be detrimental results from this type of technology utilisation. Research suggests that a structured computer environment such as those embodied in drill and practice games can encourage competition and discourage co-operation between children (Clements et al.1993) and stifle creativity (Haugland 1992). Additionally, the computer tasks provided in this software are usually out of context with the child's environment and divorced from the classroom curriculum. Finally, this utilisation of technology does not reflect the importance technology demands in our society. The majority of our schools are not meeting the challenge of the technology revolution.

Clements et al (1993) suggest that computers in education stands at a cross-roads. The first path is well worn and leads along the concept of the computer isolated in the classroom , often a reward for early finishers. The second utilises the same drill and practice and structured games, integrated however, as part of the curriculum. The third path is the more challenging and the “road less travelled”. This option means that the teacher uses the computer to liberate the curriculum. This may include using problem-solving environments to extend children's thinking, production tools as a further medium for expression and publishing (both electronic and paper) to reach extended and real audiences. This path alone offers the potential of true educational innovation.

The Geo-Logo environment is grounded in the philosophy of this third option. It aims to facilitate children's mathematical investigations and problem solving through a supportive, engaging and meaningful context. Geo-Logo - Turtle

Paths[1] is the software integrated into a complete curriculum unit that promotes geometry discovery, problem solving and mathematical application.

The curriculum unit

The research project described in this report focused on one curriculum unit from the *Investigations in Number, Data and Space* project.

The Investigations in Number, Data and Space curriculum was developed at TERC (Technical Education Research Center) in collaboration with researchers at Kent State University and the State University of New York at Buffalo U.S.A.. It is a mathematics curriculum for K - 5 that embodies an approach of active exploration of mathematical concepts.

The Turtle Paths unit is designed to investigate two dimensional geometric concepts and is the eighth unit in the Year 3 materials. The unit is substantially based around the Geo-Logo computer activities, however it includes both on and off computer tasks. A detailed teacher's guide, which includes activity descriptions, classroom management ideas, examples of dialogue that may occur and masters for student worksheets is provided with the unit.

The Geo-Logo environment

Geo-Logo is a version of Logo developed specifically for the Investigations curriculum. This computer environment was based on research that has identified both the strengths and weaknesses of Logo as a tool for learning (Clements & Meredith, 1993). Clements (1993) stated that four research-based principles guided the design of the *Geo-Logo* environment:

1. Encourage construction of the abstract from the visual.
Original versions of Logo helped children to develop mathematical strategies and conceptions out of their initial intuitions and visual approaches. For example, there is a range of research that highlights childrenÕs difficulty with turns and angles (Mitchelmore, 1993). Children can build more robust ideas of these concepts using Logo because they give turn commands and receive feedback. However, if children's Logo experience is not mediated, they can maintain misconceptions (Hoyles & Sutherland, 1989). *Geo-Logo* provides several measurement tools; for example, an on-screen protractor, placed at the turtleÕs position and heading, measures turn. One arrowhead shows the turtleÕs heading while the other follows the cursor, which students move with the mouse. When they click, this arrowhead "freezes" and a turn command is displayed. Rulers and other measurement tools are also available. However, even if students do not adopt higher order strategies and continue to use visual and empirical strategies, the Geo-Logo environment will continue to support

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their activity. The visually oriented measurement tools allow students to approach task in a wider variety of ways.

2. Maintain close ties between representations

The nature of programming creates the need to make relationships between symbols (code) and drawings explicit. However, in extended programming projects, children may lose the psychological connections between the two modes. In Geo-Logo, students enter commands in "immediate mode" in a command window. Any change to commands in either location, once accepted, are reflected automatically in the drawing. The dynamic link between the commands in the command window and the geometry of the figure is critical. Further more, the structure of the command window and its placement adjacent to the graphics window permits the immediate inspection of more commands, which facilitates connecting symbols and drawings, as well as pattern searching.

3. *Facilitate examination and modification of code and encourage procedural thinking.*

The environment supports easy creation, alteration, and use of procedures and highlights procedural-conceptual connections. The dynamic link also means that all commands in the command window represent a proleptic procedure. These commands can also be defined as a formal procedure with a tool. A "Step" tool allows students to step through commands to find errors or explore mathematical properties. Other palette tools allow easy editing and erasing.

4. Facilitates children's learning of mathematical ideas

In several other ways, Geo-Logo encourages students to build solid ideas about mathematics. For example, the Turn Rays option shows rays during turns. If the child types 'rt 120,' a ray is drawn to show the turtleÕs initial heading. Then as the turtle turns, another ray turns with it, showing the change in heading throughout the turn. A ray also marks every 30° of turn. Geo-Logo also provides co-ordinate grids, scaling of distances, geometric motions via menu, mouse control, and textual commands (including slides, flips, turns and scaling).

Structured around the Geo-Logo environment are a variety of activities that are designed to facilitate mathematical learning. These activities are integrated fully into a coherent, innovatory mathematics curriculum project: The Investigations in Number, Data, and Space project. Consequently the benefits of Geo-Logo go beyond learning geometry. Geometric models are critical for understanding numbers (number lines, multibase blocks), fractions, statistics, and other topics in mathematics as well as other subjects. In a similar vein, Geo-Logo investigations integrate many mathematical topics from mathematics and other subjects into Geo-Logo's geometric setting.

Research Implementation

The project consisted of the researcher trialling the Turtle Paths curriculum unit with a class over a period of eight weeks. Fourteen children, nine boys and five girls, from a Year 3 class in suburban primary school participated in the study. The project was conducted in the first semester of the year and at the commencement of the project the mean age of the children was seven years and nine months, with a range from seven years, four months to eight years, five months. The research site was located in the back of the children's regular classroom.

The children were assigned to pairs for the purpose of the research, on the basis of their performance on the Coloured Progressive Matrices (CPM) test as it was considered to be important that children of similar intelligence be paired to prevent one member dominating the interaction. Additionally the children were paired in gender groups - boys, girls and boy/girl. The allocation of pairs according to CPM scores resulted in four boy pairs, two girl pairs and a mixed gender pair.

Some examples…

During the course of the implementation many moments of mathematical significance were noted. The second computer task “Feed the Turtle” was very important as it introduced the concepts of turn and degrees to the children. This was especially interesting because the applicable curriculum documents provided by the state education authority comment that “it is not advised that the measurement of angles in degrees be undertaken at this level”(Department of Education, Qld, 1991, p. 234).We were therefore curious to observe the children working with this concept.

Feed the Turtle

Feed the Turtle was a maze-type task and the metaphor of a pond with 'turtle berries' was used for the children to direct the turtle around a network of paths with instructions to get the turtle to "eat" i.e. land on all of the berries in the pond. A concept of energy was used, however in this game, this factor was not a critical concept as the turtle could regain energy by eating turtle berries (Figure 1).

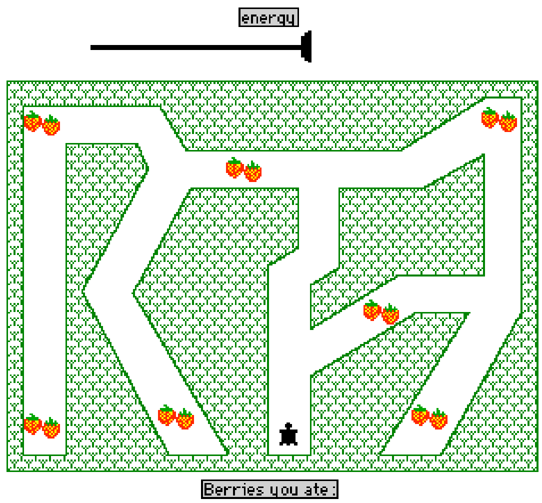

Figure 1. Feed the Turtle: A path task incorporating turns.

When this task was introduced to the group, the children identified that, unlike the first task, this maze had turns other than "square turns" (90°). This activity of course, was introducing the children to angles and the measurement of angles in degrees.
Although the concept of turning other than 90° was new, the children were keen to experiment. The first turn required in the path to the first object (or turtle berry) was 60°. The group suggested straight away that the turn must be a number less than 90° and proceeded to offer suggestions such as 50°, 45° and 70°. While it may have been mathematically appropriate to try some of the suggestions until a possible solution was reached, this strategy was not implemented in this instance. There were several reasons for this action. Firstly, this activity was designed to support the introduction of only turns of 30°. Additionally it was important at this stage to introduce the measurement of turns. Another relevant issue was that while trial and error is a valid strategy in a Logo environment, it represents a relatively low-level of problem-solving (Cope, Smith, & Simmons, 1992) that may not be optimum in a teacher-modelled interaction.

Instead, it was suggested to the children that it would be more efficient to measure a turn specifically and the “turtle turner” was introduced and demonstrated. This was the name given to a protractor-type piece of clear plastic marked with 30 degree intervals and a central turtle icon. The children utilised the turtle turner by aligning the central image of the turtle with the turtle on the screen and then reading off the required turn from the rays on the turner. At this stage too, the unit of measurement (degrees) was reinforced and the children were encouraged to use it in reference to turns. The children seemed to be receptive to these ideas and were very excited about being issued with turtle turners. In the next three work sessions they practised using these devices during the Feed the Turtle computer activity.

The use of turtle turners seemed to alleviate any hesitation of turn prediction. The children simply aligned the turtle turner with the turtle and read off the turn.

At this stage it seemed that it was possible that the use of this device may inhibit understanding of angles because it automated a process that the children had not conceptualised through construction. Furthermore it seemed that dependence on this tool may interfere with the estimation of distance established in the previous activity. It was noticed that some children at the computer attempted to use the Turtle Turner when they were working on a forward move. One even seemed to think the forward moves should now be restricted to multiples of 30. However, the children soon realised that the turner was useful for turns only and, by the end of the activity it was observed that most of the children were estimating the turn without using the turtle turner. Rather they only using the device to check the angle if they were uncertain or there was a disagreement on the amount of turn.

In general, the children were highly motivated by Feed the Turtle and from their conversation it appeared that they found it challenging. The task was quite long and a characteristic of this task was that the path regenerated after a move had been adjusted. Although some children found this frustrating because this process took time, this re-creation of the moves provided a good opportunity to analyse the progress made so far. Alison and Dana were dismayed when an error forced a regeneration of the maze near the end of the task.

Dana: Why did you put that? Now it has to start again.
Alison: Oh...Drat!
Together: fd 60, rt 60, fd 90 fd 10....
The girls chant the moves as the screen re-draws
Alison: Hey look you can just count those big lines on the turns.
Researcher: What do those big lines mean?
Alison: 30° turns... see 30, 60.
Dana: That was the longest straight bit... fd 120.
Alison: It's almost there.
Researcher: O. K. Watch carefully. What do you think?
Alison: I know it will be 60! That big one was 120, it was 4 big lines, and this one is half that size - 60!

The re-generation of the screen allowed the girls to reflect on the parameters of the Feed the Turtle and, in this instance, it provided Alison with a visual prompt for calculating the amount of the turn.

During subsequent activities the children were required to use the concept of turn and degrees in the course of their work. This notion seemed to be well grounded, as the children successfully applied the idea in several different situations. This observation was also supported by post-testing that revealed that all of the children could identify degrees as a measure of turn and two thirds of the children managed to successfully estimate the turns of three different angles.

Geo-Logo Project
The last task in the sequence was for the children to design a Logo project of their own choice. While the decisions of design and construction was ultimately

with the pairs of children, they were encouraged to follow the planning strategy used for "Geo-face", a previous activity where they created a robot face from squares, rectangles and equilateral triangles.

The boy/girl pair of the group, Tom and Claire, were the only children who did not develop of collaborative project. While Tom and Claire had worked well together during the research implementation, it seemed that their individual interests defied compromise in this instance. Despite heavily supported negotiation by the researcher to find a common interest (preferably gender free) the pair was unable to find a project that they could agree on. Consequently they were allowed to design a project each. Tom drifted towards other pairs working together and eventually designed and produced (with help from various other children) a speed boat similar to several of the other projects. Claire on the other hand, knew exactly what she wanted to do and set about drawing a human house (Fig 2).

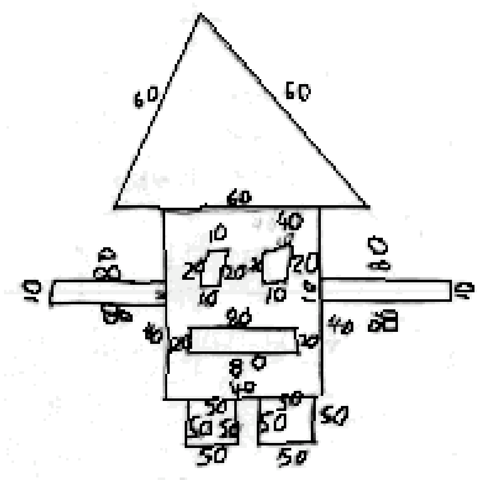

Figure 2. Claire's human house.

This design was based quite heavily on the Geo-face activity while the drawing fairly simple, it was very suited to convert to Logo commands.

Although Claire's design was not remarkable in itself, the production process that she used was meticulous. While the other children started with a sketch, converted the drawing into parts and then wrote a few procedures that may have been useful when they moved to the computer, Claire planned every step of her project from the start. Her design was made up of rectangles and triangles and each part in the drawing was carefully labelled (Figure 2). She then used the planning matrix (from geo-face) to write commands for each of the shapes. For each object she determined a perimeter, calculated the turns she required and then wrote the appropriate procedure. Then, when she moved to the computer all she needed to do was type in the planned commands.

The engagement Claire demonstrated with this task was salient. While she had some assistance with the dimensioning process of her drawing, the majority of the work was performed independently. At the computer she worked methodically through the procedures, often working into recess time to finish. Each procedure was tested independently and then the picture was assembled.

While much of the conceptualisation had been completed off-computer, Claire found a major flaw in her plan when she constructed the components. She started with the eye and then the mouth, which were both to her satisfaction. The next component she created was the body that, at 40 by 40, was visibly far too small (The mouth was 80 across). At this stage Claire consulted the researcher, not to ask for help but rather to seek permission to deviate from her plan.

> *Claire:* This was going to be the body but it's too small. It would be good for a foot but.
> *Researcher:* O. K. Do you want to change the name of that one to foot?
> *Claire:* Yes.
> *Researcher helps Claire locate the procedure in the teach window and change the name to foot.*
> *Researcher:* So now what will you do?
> *Claire:* I need a really big body. The mouth is about that big (*uses fingers to calliper on screen*) so the body has to be about that big. The mouth was 80 so the body should be um. . . 100?

At a later stage Claire also adjusted the roof (an equilateral triangle) from 60 to 120 so that it would fit with the new body size.

This vignette illustrated some pertinent issues for the research. Firstly, even though Claire had not demonstrated complete understanding for any of the mathematical concepts presented in the prior mathematical test, in this instance she was posing and solving her own mathematical problems very successfully. Claire's success as a mathematician seemed to stem from two key factors provided by the circumstances. First of all she had a real purpose for the application of mathematical principles and secondly she was engaged and focused with the task at hand.

Additionally, although the purpose of this project was not to teach the children how to program in Logo, Claire was in fact demonstrating attributes of an accomplished computer programmer. Her approach could be equated to a "top down" programming style in which she defined her goal and then broke it down into manageable components. She also revealed cycles of control in which she designed, tested and then re-appraised, modifying where necessary.

It was evident from this scenario that Claire encountered some rich and valuable mathematical learning during the activity. The nature of the task and the computer environment most certainly facilitated this experience, however it is important to examine any other factors influencing her engagement with the task.

The teacher's role in the construction of knowledge

Although this research project has indicated that children may thrive in this type of learning environment, the limitations of the situation were obvious. The most restrictive feature of the project related to the teacher's role in the curriculum implementation. Even though the Turtle Path Teacher Book (Clements et al, 1995) offered guidance for implementation and some class discussion, the decision of when and how to intervene in the learning process was largely dependent on teacher intuition. While it is established that, according to constructivist principles, children construct their own mathematical knowledge, it seems that the constructivist perspectives of teaching are less defined. (Simon, 1995). How can a teacher ensure that children are provided with the necessary conditions to activate the construction of knowledge?

In terms of the Geo-Logo curriculum, it seems that the key to children constructing their own understandings of the concepts lies with mechanisms of teacher support. During this implementation, the support of children working with the tasks was dependent on the researcher's perceptiveness. In the course of the study, intervention included mediation between pairs, encouragement to spend more time investigating a concept, help with "thinking through" a concept, assistance with the technical aspects of using the computer environment and moral support when the task seemed too difficult to continue.

However, it is often difficult to ascertain whether it is appropriate to intervene in the learning process. During analysis of video-tapes it was far more simple to identify moments when it seemed that children needed support than during the bustle of a classroom environment. Likewise it was evident from the video-tapes recorded during this research that sometimes assistance was provided too early to children who would have benefited from further independent exploration. What are the conditions that make it appropriate to intervene? And when should this scaffolding be withheld?

Further Research

Further research is warranted to examine the nature of teacher support during on-computer exploration tasks. It appears that broadly, the make-up of these support mechanisms is social, emotional, cognitive or technical, however it is important that the specific characteristics of intervention are identified. Furthermore, it is anticipated that an important component of this investigation will be examining the scaffolding mechanisms of experienced teachers who regularly support children through spontaneous intervention in active exploration situations. Ultimately, further research should investigate which facilitation strategies are appropriate to provide optimum investigation in computer-based learning experiences. It is anticipated that understandings of this nature will lead to more constructive learning experiences for children exploring with technology based learning opportunities.

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