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Paper Title: **Promoting Metacognition in Developmental Math Students**

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Abstract: This session presents efforts to develop metacognitive skills in mathematics learning. These skills are seen in current research not only as necessary conditions but also critical for learning mathematics.

Current research in the area of metacognition and mathematics education supports the belief that metacognitive skills are a needed condition for learning mathematics. Metacognition is viewed as important not only in problem solving but in all mathematics performance. The study of metacognition argues for a change from the traditional, teacher-centered instruction to instruction addressing cognition metacognitively. Examples of this new educational trend include cooperative learning activities, the promotion of reflective thinking through discussion and writing, and the development of self-regulation.

In an effort to create self-awareness and promote independent learning in their mathematics learning, students can be actively engaged in their learning by whole class or small group discussions, writing activities, and self-monitoring tasks. In an effort to identify misconceptions of these methods currently in use in the mathematical setting, and the belief that these methods are not applicable in mathematics, this presentation will describe the activities used in each of these categories.

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Promoting Metacognition in Developmental Math Students

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Relation to the Third International Seminar on Misconceptions in Science & Mathematics

This session presents efforts to develop metacognitive skills in mathematics learning. These skills are seen in current research not only as necessary conditions but also critical for learning mathematics.

Current research in the area of metacognition and mathematics education supports the belief that metacognitive skills are a needed condition for learning mathematics. Metacognition is viewed as important not only in problem solving but in all mathematics performance. The study of metacognition argues for a change from the traditional, teacher-centered instruction to instruction addressing cognition metacognitively. Examples of this new educational trend include cooperative learning activities, the promotion of reflective thinking through discussion and writing, and the development of self-regulation.

In an effort to create self-awareness and promote independent learning in their mathematics learning, students can be actively engaged in their learning by whole class or small group discussions, writing activities, and self-monitoring tasks. In an effort to identify misconceptions of these methods currently in use in the mathematical setting, and the belief that these methods are not applicable in mathematics, this presentation will describe the activities used in each of these categories.

INTRODUCTION

Studies in the last decade have indicated that mathematics education is in a state of crisis. For example, a cross-national survey of high school seniors' achievement in algebra and calculus reported that US students scored well below their peers from other international countries. In another similar study in the mid-sixties, US students had scored near the median. This dramatic decline in the placement of US students in mathematics achievement within the last two decades is very disturbing. These students were chosen among the 95 percentile of the high school senior population, the most likely and probably college bound students (Savage, 1986).

To add to this concern, many other studies have attributed this decline in achievement of US students to deficiencies in problem-solving and higher-thinking skills (Savage, 1986). These skills need to be taught and developed within the classroom, and adequate training through workshops and seminars must be provided to instructors. An emphasis that covers the spectrum of cognitive strategies which can be easily incorporated into the curriculum must also be provided and supported. Some of these strategies and their implications in mathematics education, particularly developmental mathematics, are briefly discussed in this paper.

WRITING TO PROMOTE METACOGNITIVE THINKING

Traditional views hold that students learn to write in other areas of the curriculum and not in mathematics. Mathematics is traditionally viewed as only computational. The belief that writing be used in the mathematics classroom has been given little thought in the past. In recent years, however, this position has been changing, and different studies have recommended increased writing about mathematical concepts and applications by students as a useful and valuable aspect of learning mathematics (Burton, 1985; Greenius 1983; Johnson, 1983; Shaw, 1983; Watson, 1980).

One of the ways students can acquire new information is through forming ideas into writing. Both Emig (1977) and Haley-James (1982) have expressed writing as a mode of language that lends itself to the processing of new information. Through the use of writing, mathematics instructors can cultivate many thought processes embedded by their students. Processes such as analyzing, reflecting, and synthesizing of material are required in a precise way. Johnson (1983) suggests that if students can write clearly about mathematical concepts, then they probably understand them. The implementation of writing in the classroom is viewed by many as beneficial to the instructor as well as the student (Rose, 1989).

Developing metacognitive thinking in developmental math students through the use of writing in the classroom has been viewed as an instructional aid and as a learning tool for the developmental math student. One of the most valuable tools a student can develop is the ability to communicate and retain mathematical knowledge. Although the use of writing in the classroom may be a little unusual, students should be expected to process ideas and concepts through writing in the mathematics classroom. The usage of mathematical terminology, sharing of attitudes about the concepts, developing an understanding of how to summarize concepts, and explanations of error patterns, occur in the actual writing processes. Since past traditional methods have not proven to be effective for many students, recent approaches using writing must be explored. The use of writing can be seen as effective in developing the student's ability to shift from the rote style of learning into the next level of understanding and applying the meaning of concepts. This should be a goal for the instructor as well as the student.

Past researchers have successfully used journal writings (Nahrgang & Peterson, 1986; Borasi & Rose, 1989) to get students to vent anxious feelings about mathematics and the problems encountered in learning concepts. Bell and Bell (1985) found expository writings as effective and practical tools in problem solving. Impromptu writings, cited in this paper, have provsecondary mathematics teachers (Miller & England, 1989).

Instructors construct the impromptu writing prompts to solicit student responses to a specific problem, situation, or question. The writings could be used in the areas to address self-awareness and self-regulated learning, habits of study times, attitudes and misconceptions about mathematical concepts, and most importantly, the understanding of concepts. A limited amount of time is given to the student to read the prompt, formulate thoughts, and write their response. Students are asked to provide clear expressions of their understandings related to the prompt. Although more often students are asked to write at the beginning of the period, an alternate form of this would be to have the students write at the end of the period. The use of writing at the beginning of the period allows the setting to become an immediate and involved time. When writing is used at the end of the period, this provides an excellent activity for bringing closure to the concept being discussed.

The summarizing of these writings can be quickly viewed by the instructor and later reviewed more extensively. The advantages of writing as beneficial for the instructor and the student will be addressed later. The writings used in this paper included areas to address self-awareness and self-regulated learning, habits of study times, attitudes and misconceptions about mathematical concepts, and most importantly, the understanding of concepts. It is important to note that extra time is involved in the reading and reflection of the student writings. Additional time may be needed if the instructor chooses to react to the writings but this could prove beneficial for the student to see that the instructor demonstrates the importance by responding. This process can be achieved at quicker rates after some time. Benefits from the use of writing for the instructor will quickly be noticed, and the actual time spent responding will develop the necessity to use the writing.

The following is a partial listing of the sample prompts used to incorporate writing in a developmental mathematics classroom:

1. When you work a math problem, how do you determine if your answer is correct? When you think an answer is incorrect, what do you do?
2. Pick a homework problem from page 85 that you thought looked difficult but you think you worked correctly. Explain why you think your work is correct. Pretend that you are writing for someone who does not understand algebra.
3. What type of equation do you find the hardest to solve? (Use page 143 to give an example). Explain what you think makes this problem difficult for you.
4. How do you know when you have solved an equation correctly?
5. How do you know if an expression has a common factor? Describe your decision-making in detail.
6. What type of factoring was the most difficult for you? Explain why you think you have trouble with this problem.
7. Give an example of the sum of two squares and the difference of two squares. Explain how you would determine if either or both of these can be factored.
8. Division by zero is undefined in mathematics. What does this statement mean to you? How well do you understand this concept?
9. How do you determine if a rational expression can be simplified? If an expression can be simplified, how would you do so?
10. Please, explain why the following problem is correct or incorrect.

Benefits of writing are supported by feedback from student writings which included comments such as "I thought I understood the problem until I was asked to write about it." Other comments included "I am not sure about the problem, but I think, etc." In addition to error patterns and common misconceptions that were discovered through the review of writings, the following instructor benefits emerged:

1. Allowed the student to view math in a less threatening way.
2. Allowed the freedom to communicate privately and individually.

3. Allowed the student to prepare before class.
4. Allowed the student to use writing across the curriculum, not necessarily in quality but informally.
5. Increased student awareness of mathematics.
6. Allowed the student to see the need for note-taking and reading of text.
7. Strengthened study skills and therefore, more focus on learning.
8. Allowed a venting of anxious feelings and a release of fears through writing.

In the early stages of writing the responses were limited in the amount of writing, and these first responses were important in that they established a model for what would continue in the next few weeks. As the semester continued, the student writings improved in quantity and in quality. While earlier writings were often vague, later responses became more specifically focused. Initially, there was some reluctance to write. Eventually, student opposition ceased, and the students seem to enjoy the "free" writings. Responses were of higher quality when they could write to someone such as a classmate, friend, or to the instructor. Also, simple and more direct prompts developed higher quality. Attitudes and dealings solicited more responses than discussions that centered on concepts, operations or direct problem-solving.

As mentioned earlier, valuable benefits were observed and misconceptions were addressed. Students wrote about concepts they understood or did not understand. This was an effective tool in helping the instructor to alter instructional methods. Over the course of the semester, attitudes between the instructor and the student proved to be a positive and close relationship. The students felt it was important to vent feelings and misconceptions about mathematics and the instructor felt it was important to find ways to overcome these feelings. An overall reward for all instructors would be to gain insight into the teaching of mathematics through the use of student writings which could lend to improved and effective instruction. In addition, if student attitudes about mathematics were improved, better academic performance would be a reality.

PROMOTING METACOGNITIVE THINKING THROUGH SELF-REGULATED LEARNING

Class time is precious. Often, instructors are hesitant to introduce support material because it may take away from teaching time. In this area, self-regulated learning can be implemented through a concept-checklist of major concepts discussed throughout the course. After each major testing period students can compare the missed items on the test to the list of concepts. This can provide immediate feedback of misunderstood concepts. All too often, students categorize lost test points as "careless" errors. After careful examination of several tests, students will often find that points are deducted on Test 1 for the same reason points may have been deducted on Test 2, Test 3, etc. The notion that careless errors occurred on all of the tests can quickly be disregarded. Also the use of this checklist on successive tests can establish the self-awareness that a misunderstanding of concepts exists and self-monitoring of skills can be acquired. At this time the student can institute the need for reviewing of concepts missed on tests. This systemizes the student's ability to later retain the misunderstood concept in an understandable manner that is accountable. Often times, students do not realize that a concept is misunderstood until tested on it. An additional tool can be added by having the student calculate the total points lost over several testing periods to support the necessity to analyze and prepare for modifications for these misunderstood concepts. This development lends to an increase of self-regulated learning which could give rise to more effective communication and interminable retention within the mathematical classroom. Listed below is an abbreviated version of the self-checklist. It is intended that all major concepts be listed and students complete the rows for missed test points to visibly monitor lost points.

An additional tool to increase student awareness is to employ the use of an error analysis form. An account of demonstrating the missed problem on the test is repeated, and then the student demonstrates the problem corrected, and an added explanation of why the problem was missed in words. In addition, students can be asked to write about why they lost points on tests and the listing of what factors contributed to the deductions. Also, having them to identify what changes they plan to make for future testing may also be incorporated to embed the metacognitive strategies. Listed below are the two tables discussed above which can be incorporated to develop self-regulated and self-monitoring.

Student Worksheet - Analysis of Errors

Problem as originally worked on test	Problem as corrected	Explanation of error

The above table is a sample of the format used in the error analysis. Students are actively engaged in an accountability of cognitive assessment. Often times tests are put away without assessing why the problems were marked incorrect. Through the use of this table, the students are also forced to make attributes for both successes and failures. The instructor can also guide students to reinforce internal and controllable attributions such as efforts and strategies. Uncontrollable, external factors, such as luck and task difficulty are also examined. The need for adaptation and change when the performance is not satisfactory is also communicated and monitored.

Self-Check List

Concept	Test One	Test Two	Test Three	Test Four

The examination of lost test points forces students to identify specific strategy repertoire. Suggestions for students to realistically monitor their performance can modify goals, both short-term and long-term for making plans to repeat the course or enroll in the next course. Also, changes in the course material and perhaps as more difficult concepts develop, students can account for strategic studying habits. The identification of students with a sensitivity of cognitions, a repertoire of strategies, and the conscious monitoring of the student's independent role as a "math" student can also be monitored through the chart above.

COOPERATIVE LEARNING STRATEGIES TO PROMOTE METACOGNITIVE THINKING

Helping students not overcome attitudinal, emotional, and timely buildup of obstacles within the classroom is most challenging. The support from the instructor to overcome these obstacles is often not in itself sufficient. Allowing students to work in small groups can have many benefits. Using small group learning as a cooperative setting as opposed to a competitive setting, allows a more enhanced learning and at the same time a higher self-esteem. Enhanced learning can be achieved through the more confident student in being able to actively participate in explanations of concepts. The building of self-esteem can be achieved through the less confident student being able to feel comfortable having the concept explained in a small setting. The shared accountability of students within a small group setting can be a motivational tool also. The accountability factor within the small group setting can justify for more consistency in attending class also. The verbalizing of mathematical concepts, the usage of mathematical terminology, and the development of listening skills can certainly build metacognitive thinking skills and establish foundations for critical thinking. The ability of students to formally explain concepts rather than demonstrate rote memorization can lead to increased vocabulary and communication between students in the small group setting and build for higher and independent learning. Traditional methods have proven to be ineffective in preparing students for sequencing math courses. Further testing and research can be studied to examine the achievements of cooperative learning. Customizing small groups early in the classroom can lessen anxiety and establish shared attitudes and emotions. The monitoring provided by the instructor can also help lessen these anxieties and emotions that can often hinder maximum learning. Small group activities can include problems from the textbook, problems from the instructor, or problem that students may have questions about. An alternate form would be to incorporate "group" quizzes. It is most important that to establish a relaxed setting, the groups can be randomly assigned, and often altered to form new groups. An alternate of cooperative learning is to have the small groups go to the blackboard or the overhead to show the actual solving processes of problems. The use of a four step problem could have each student completing a step to the problem and each of the other group members sequencing the process by each having a turn in a cooperative manner. It is important to note that this type of relaxed setting must be formulated and developed through the sensitivity of the instructor.

CONCLUSION

There are many advantages to classroom research. I learned that first of all that what I believed to be intuitively true, that is developing metacognitive thinking skills through writing, cooperative learning, and self-monitoring activities can help students learn more mathematically. I also found that through the use of writing, a great assistance for those who need it the most. Other spin-offs were also noticed.

Now when I begin new concepts I am trying to incorporate pretesting and posttesting so that student progress can be studied. It was at one time most disturbing if a student did not perform well on a unit test after all of my wonderful teaching. Instead I now recognize that a student who scores 62% on a unit test has made progress when compared to a 35% on earlier testing involving the same concepts.

I now view myself not only as "just a teacher," but also as a "researcher and assessor" using the classroom as a laboratory. Teaching is discovering effective strategies that produces successful students. Improved attitudes from myself and from students have also increased. Rote and routine math exercises have been deleted and experimentation to include everyone as learners has been incorporated.

An effective instructor in developmental mathematics must be supportive, respectable, encouraging, patient, and challenging. Helping students with cognitive and emotional difficulties in the classroom is not an easy responsibility. In traditional classrooms settings with whole-class instruction can still be effective for many students. However, for many students it has proven to be ineffective. The incorporation of teaching styles that can actively involve student participation is not only wise but essential to increase metacognitive thinking to allow student progression. This can be achieved through the use of writing in the classroom with responses monitored by the instructor, the use of a concept checklist, incorporated with the error analysis worksheet, and the promotion of cooperative learning in small group settings, can certainly support the overcoming of students with cognitive and emotional obstacles. Through the use of these strategies students are encouraged to express and reflect upon their attitudes, knowledge, processes, and beliefs about mathematics. The self-assessment of their own understanding of the content and skills being taught and learned can perhaps be encouraged to reconstruct their views about mathematics. It is hoped that through these strategies, the exposure of individual needs and difficulties as well as the provision of immediate feedback to their instructor can account for independent and higher learning. Hopefully, as teachers employ these strategies, they can become more responsive and effective in their instruction. Perhaps, eventually the reconceiving of educational beliefs could prove to have long-term effects, and could assist students in building confidence and competence in the field of mathematics. It is hoped that through this paper students will have more opportunity to acquire basic skills in the developmental mathematics classroom in the employment of these nontraditional strategies.

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