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Misunderstandings of Kinematics Graphs

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ABSTRACT

Recent work has uncovered a consistent set of student difficulties with graphs of position, velocity, and acceleration versus time. These include graph as picture errors, slope/height confusion, problems finding the slopes of lines not passing through the origin, and the inability to interpret the meaning of the area under various graph curves. For this particular study, data from 895 students at the high school and college level was collected. Individual test items were examined to reveal common difficulties. The test as a whole should prove useful for other researchers studying kinematics learning as well as instructors teaching the material.

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INTRODUCTION

A considerable effort has been made to examine what physics students learn from their introductory classes dealing with kinematics–the motion of objects. Although it is not clear why this one area of physics instruction has received more attention than others, one might speculate that researchers have recognized the importance of this topic as a "building block" upon which other concepts are based. Alternately, teachers might hope that since kinematics is almost always taught early in the curriculum, helping students grasp these ideas will give them the confidence to approach the rest of the course with less anxiety. A more pragmatic consideration is that the early availability of microcomputer-based labs which allowed real-time measurement of position, velocity, and acceleration held the possibility of drastically changing the way these concepts could be taught. Researchers were interested in knowing if the new MBL approaches to teaching were viable. Regardless of the reason, it is now quite easy to find many studies of student alternative conceptions in kinematics. Hestenes, Wells, and Swackhamer's (1992) Force Concept Inventory and Hestenes and Wells (1992) Mechanics Baseline Test are excellent assessment tools based on this earlier work. Unfortunately, there is less research on students' problems with the interpretation of kinematics graphs. This project was an attempt to replicate those few existing studies, find additional difficulties if they exist, and develop a useful research tool for others interested in working in this area.

WHY GRAPHS?

The ability to comfortably work with graphs is a basic skill of the scientist. "Line graph construction and interpretation are very important because they are an integral part of experimentation, the heart of science." (McKenzie & Padilla, 1986, p. 572). A graph depicting a physical event allows a glimpse of trends which cannot easily be recognized in a table of the same data. Mokros and Tinker (1987) note that graphs allow scientists to use their powerful visual pattern recognition facilities to see trends and spot subtle differences in shape. In fact, Chambers, Cleveland, Kleiner, and Tukey (1983) state that there is no other statistical tool as powerful for facilitating pattern recognition in complex data. Graphs summarize large amounts of information while still allowing details to be resolved. The ability to use graphs may be an important step toward expertise in problem solving since "the central difference between expert and novice solvers in a scientific domain is that novice solvers have much less ability to construct or use scientific representations." (Larkin, 1981, p. 121)

Perhaps the most compelling reason for studying students' ability to interpret kinematics graphs is their widespread use as a teaching tool. Since graphs are such efficient packages of data, they are used almost as a language by physics teachers. Unfortunately, this study indicates that students do not share our vocabulary.

KNOWN PROBLEMS

Physics teachers often report that their students cannot use graphs to represent physical reality. The types of problems physics students have in this area have been carefully examined and categorized by Barclay (1986), McDermott, Rosenquist, and van Zee, E., (1987), Mokros and Tinker (1987), and van Zee and McDermott (1987). Several of these studies have demonstrated that students entering introductory physics classes understand the basic construction of graphs, but have difficulty applying those skills to the tasks they encounter in the physics laboratory.

Kinematics graphs have position, velocity, or acceleration as the ordinate and time as the abscissa. The most common errors students make when working with these kinds of graphs are: (1) thinking that the graph is a literal picture of the situation and (2) confusing the meaning of the slope of a line and the height of a point on the line (Barclay, 1986; Mokros & Tinker, 1987). The first of these might occur when a student is asked to draw a velocity versus time graph of a bicycle going downhill , uphill, and then on level road. Many students produce incorrect velocity graphs which look like the hills and valleys traversed by the bicycle. It is easy to see how the path of the bike is mistakenly taken as a cue in drawing the graph. In another situation, students asked to find the point of maximum change in a graph sometimes indicate the point of largest value. These types of errors may indicate that students view lines on these graphs as something concrete rather than as an abstract indicator of trends.

In general, students tend to find slopes more difficult than individual data points (Price, 1974). They also have a hard time separating the meanings of position, velocity, and acceleration versus time graphs (Halloun & Hestenes, 1985). Regardless of the type of errors students make, it is generally agreed that an important component of understanding the connection between reality and the relevant graphs is the ability to translate back and forth in both directions (McDermott, et. al, 1987).

Recognizing the general importance of graphing skills and the recent interest in students' interpretation of kinematics graphs leads to the need for assessment of those skills. "The construction of a valid and reliable instrument for assessing specific graphing abilities would be a step toward establishing a base line of information on this skill." (McKenzie & Padilla, 1986, page 572) The purpose of this study was to produce such an instrument for measuring the understanding of kinematics graphs.

METHODS, DATA SOURCES, AND RESULTS

The first step in this process was to formulate a list of behavioral objectives which relate to an understanding of kinematics graphs. Eight objectives emerged from an examination of several commonly used introductory physics books (Halliday & Resnick, 1978; Sears, Zymanski, and Young, 1980; Kane & Sternheim, 1978), materials from the Senior Division of The Ontario Assessment

Pool—Physics Kinematics Section (Ontario Institute for Science Education, 1981), and informal interviews with science teachers. After preliminary research, one objective was eliminated. Nearly all students were able to go from a point on a graph to its coordinate pair, and vice versa. Since I was interested in finding student difficulties, this objective was removed from later versions of the test. The remaining objectives are listed in table 1. It is important to note that no graph construction objectives are included.

Given:	The student will:	Difficulty
Position-time Graph	Deduce Velocity	0.51
Velocity-Time Graph	Deduce Acceleration	0.40
Velocity-Time Graph	Deduce Displacement	0.49
Acceleration-Time Graph	Deduce Change in Velocity	0.23
A Kinematics Graph	Select Corresponding Graph	0.38
A Kinematics Graph	Select Textual Description	0.39
Textual Motion Description	Select Corresponding Graph	0.43

Table 1. Objectives of the Test of Understanding Graphs–Kinematics. The objective difficulty values refer to the final version of the test.

Three items were written for each objective, producing a test of twenty one multiple choice questions. As noted earlier, several outside sources were useful in supplying items which were adapted for the Test of Understanding Graphs–Kinematics (TUG-K); however, most test items were original. An effort was made to ensure that only kinematics graph interpretation skills were measured. For example, an item asking a student to "Select the graph which correctly describes the vertical component of the velocity of a ball tossed into the air." would not be appropriate since it tests knowledge of projectile motion. Items and distracters were also deliberately written so as to indicate when students held commonly seen graphing difficulties. For example, if students were asked to locate the steepest slope, the point with largest ordinate value was available as a distracter.

Early versions of the test were submitted to 134 community college students who had already been taught kinematics. 96 of these students were taking a non-calculus based technical physics course. The remaining 38 were enrolled in the first semester of a calculus based engineering physics sequence. These results were used to modify several of the questions. These revised tests were distributed to fifteen science educators including high school, community college, four year college, and university faculty. They were asked to complete the tests, comment on the appropriateness of the objectives, match items to objectives and criticize the items. This was done in an attempt to establish content validity and the correctness of the answer key. The tests were also given to 165 11th and 12th grade students from three high schools and 57 four year college physics students. Again, all students had been taught kinematics previously. After each student had taken one version of the exam they were randomly assigned to one of four different graphing laboratory exercises. These exercises were approximately two hours in length. Within a week of the lab experience, they took a second version of the test. A paired samples t-test revealed a significant increase in the mean scores between pre and post versions ($t = 4.864$, $df = 221$, $p < 0.0001$). Since the graphing exercises dealt exclusively with kinematics graphs, this was seen as evidence of construct validity. A group of 15 community college students took both versions of the test with no graphing exercise between administrations. This was done to check for practice effects. A paired sample t-test indicated no significant difference between scores on the pre and posttests ($df = 14$, $p = 0.47$). (The Pearson product-moment correlation between the pre and posttest scores was 0.79, indicating that the two forms were indeed parallel.) Apparently students do not learn much about kinematics graphs just from taking the test.

A final version of the test was prepared from the most discriminating items of the earlier versions. A few new questions were added to further examine interesting patterns emerging from the preliminary data analysis. This test was given to 524 college and high school students from across the country. The results are summarized in table 2.

Descriptive Statistics
N = 524 post-instruction high school and college students
Mean = 8.5/21 = 40%
Standard Deviation = 4.6/21
Standard Error of the Mean = 0.2

Reliability Analysis
KR-20 = 0.83
Average Item Point-Biserial Coefficient = 0.74

Discriminating Ability
Average Item Discrimination Index = 0.36
Ferguson's Delta = 0.98

Table 2. Statistical results from the final version of the test.

DISCUSSION

The mean score of 40% is quite low considering that the test was taken following instruction in kinematics. The case could even be made that this instruction might be better than the norm. The teachers who administered the test to their students were volunteers. This might lead to a bias in the student population since it is possible that only good teachers would "risk" an outsider's close examination of what their students were learning. It can certainly be said that the teachers who volunteered were interested in improving instruction. But the results are clear, whether the instruction was exemplary or ordinary, the students were not able to interpret kinematics graphs.

The rest of the analysis indicates that TUG-K has construct and content validity and is a reliable test of understanding of kinematics graphs for groups of high school and college level students taking introductory physics. It is certainly useful for diagnostic purposes and should be a helpful research tool. It was found that calculus-based physics students did better on the test (with a mean of 9.8 vs. 7.4) than algebra/trigonometry-based physics students ($t = 4.87$, $df = 335$, p

< 0.0001), but college students as a whole did no better than their high school counterparts ($t = 1.50$, $df = 522$, $p < 0.13$). The mean for males of 9.5 was significantly better than the 7.2 value for girls ($t = 5.66$, $df = 491$, $p<0.0001$).

I was able to verify the findings of earlier research concerning "graph as picture errors," slope-height confusion, and lack of discrimination between kinematics variables. Students were also found to have specific difficulties calculating slopes and interpreting the meaning of areas under curves. These are summarized in table 3. Items which highlight several of these problems will be discussed below. Since the first three problems have been thoroughly covered in the literature, they will not be discussed here.

STUDENT DIFFICULTIES WITH KINEMATICS GRAPHS

Graph as Picture Errors

The graph is considered to be like a photograph of the situation. It is not seen to be an abstract mathematical construct, but rather a concrete duplication of the motion event.

Slope/Height Confusion

The highest/lowest axis value has the highest/lowest slope. Students often seem to read values off the axes and directly relate them to the slope, whether they apply or not.

Variable Confusion

Students see little difference between distance, velocity, and acceleration. They often believe that graphs of these variables should look identical. This might be related to the graph as picture error. If a graph is like a photograph, it shouldn't matter what is graphed, it will look like a replication of the object's physical motion.

Slope Errors

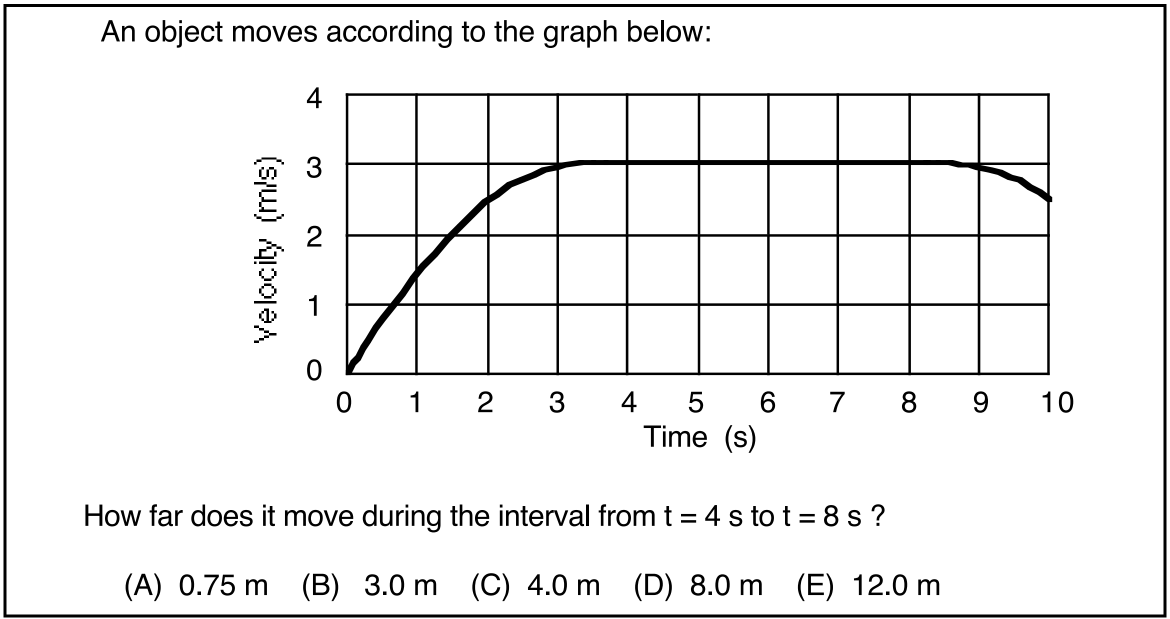
Students successfully find the slope of straight lines which pass through the origin. However, they have difficulty determining the slope of the line (or the appropriate tangent line) if it does not go through zero.

Area Difficulties

Students do not recognize the meaning of areas under kinematics graph curves. They also misinterpret the word "change" to automatically refer to slope, even though it might involve an area concept.

Table 3. Implications from an item analysis of the graphing test.

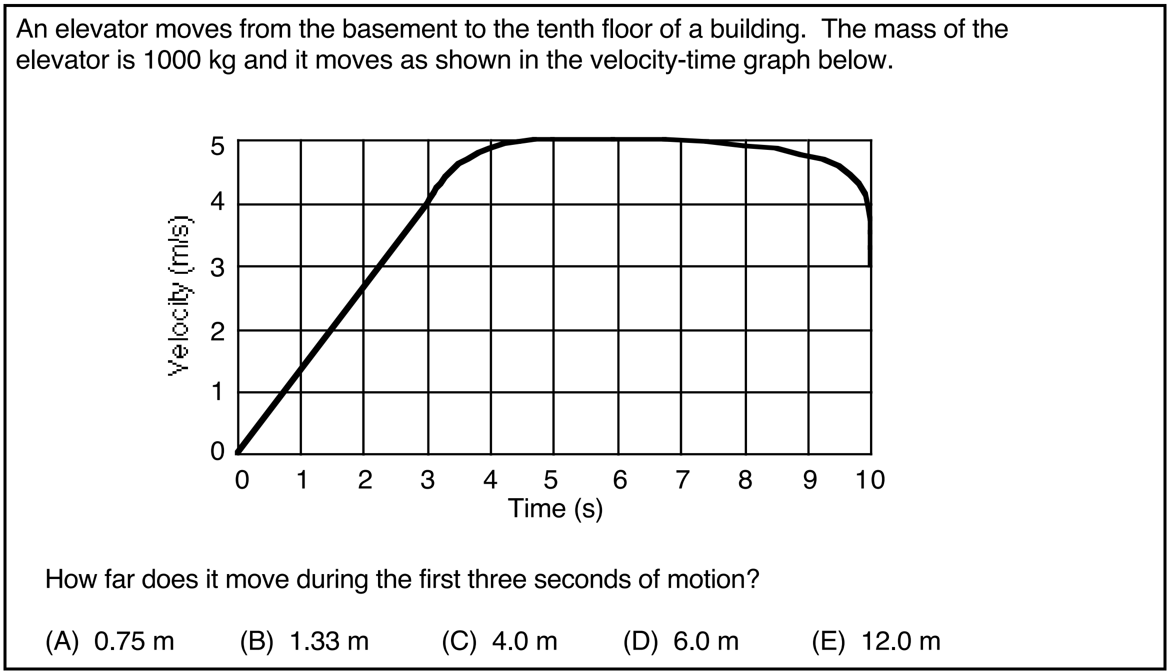
As predicted by studies noted earlier, it was found that students have considerable difficulty determining slopes. However, this research indicates that this is only true for "unusual" lines. If the line went straight through the origin, 73% were able to correctly determine the slope. In fact, a question requiring this calculation was the easiest item on the test. However, if the tangent line did not pass through the origin, item difficulties dropped to 21% and 25%, approximately the guessing level.


An object moves according to the graph below:

How far does it move during the interval from t = 4 s to t = 8 s ?

(A) 0.75 m (B) 3.0 m (C) 4.0 m (D) 8.0 m (E) 12.0 m

Figure 1: An easy item, 72% correctly answered it. Comparing to the next figure indicates that students answered this question by reading the velocity from the vertical axis and multiplying by the time interval.


An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below.

How far does it move during the first three seconds of motion?

(A) 0.75 m (B) 1.33 m (C) 4.0 m (D) 6.0 m (E) 12.0 m

Figure 2: Only 28% answered this item correctly. This indicates that students cannot interpret areas under curves, even though they may occasionally appear to be able to do so, as shown by the previous figure.

It was found that students had tremendous difficulty interpreting the meaning of areas under curves. In only one item (see figure 1) were they able to calculate distance traveled from a velocity-time graph. Item difficulty was 0.72, making this the second easiest test question. Taken by itself, this might lead one to believe that students can determine areas. However, comparison to a very similar problem (figure 2) shows that this is not the case. Item difficulty for this second item was only 0.28. Apparently, students working on the first item were determining the distance simply by reading the velocity from the vertical axis and multiplying it by the time interval—not even realizing they were finding an area! In other words, students were able to recall and use a formula ($s = vt$) to find distance covered, but could not determine the same information by looking at a graph and calculating an area.

Similar difficulties interpreting areas were found with the item illustrated in figure 3. This was the hardest question on the test. Only 16% of the students correctly selected the graph with the largest area under the curve. 41% selected choice A, perhaps being misled by the use of the word "change" in the questions. A similar item, this time asking for the smallest change in velocity elicited the consistent reverse response, only 30% chose the correct answer while 62 percent selected the item with the flattest line.

Acceleration versus time time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

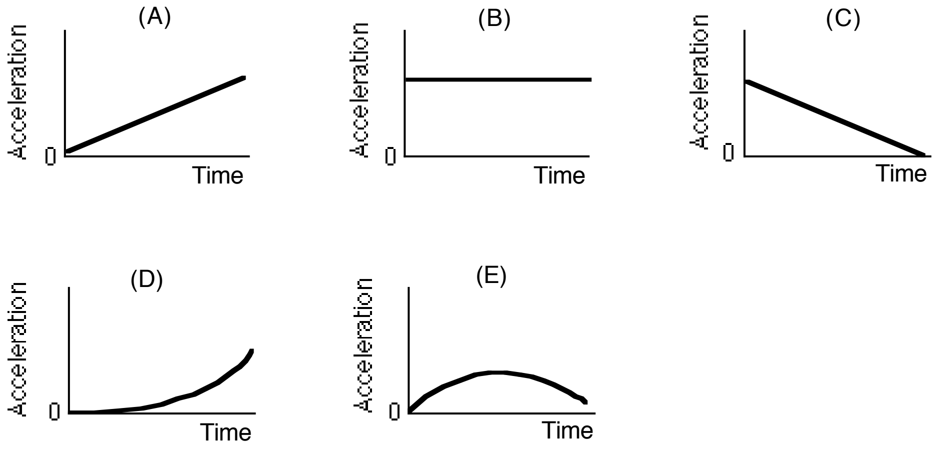


Figure 3: The most difficult item on the test. The question is really asking students to pick the graph with the largest area under the curve. Only 16% could do so.

IMPLICATIONS FOR INSTRUCTION

What can be done to address the difficulties students have with the interpretation of kinematics graphs? The first step is for teachers to become aware of the problem. Knowing that students cannot use graphs as "fluently" as they should means that discussions of the kinematics variables cannot start by just referring to their graphs. Students need to understand graphs before they can be used as a language for instruction. Teachers may want to utilize Arons' idea of operationally defining kinematics concepts (Arons, 1990). It is possible–and probably even desirable–to use graphs to help students begin to understand what the kinematics variables mean. But instruction incorporating these graphs must include thorough explanations of all the information each one relates. This study indicates that teachers must also choose their words carefully and specifically warn students that "change" does not automatically signify "find a slope."

Teachers should deliberately include motion events where the kinematics graphs do not look like photographic replicas of the motion and the lines don't all go through the origin. Instuction should require students to go back and forth between the different kinematics graphs, inferring the shape of one from another. They should also be asked to translate from motion events to kinematics graphs and back again. Finally, teachers should have students determine slopes and areas under curves and relate those values to the motion event.

All these suggestions for modifying instruction can all be summaried by one phrase: Teachers should give students a large variety of "interesting" motion situations to examine graphically.

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